Unemployment Modelled Through M|G|∞ Systems (Revisited)

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Abstract **- Here it is resumed the work presented in Ferreira, Filipe and Coelho (2014) where, using the results on the M|G|∞ queue busy period, it is presented an application of this queue system in the unemployment periods' parameters and distribution function study. Now it is completed with an evaluation of the assistance costs.**

Keywords - $M\,|\,G\,|\,\infty$, busy period, unemployment.

1. The Model

In the M|G|∞ queue system

- The customers arrive according to a Poisson process at rate λ
- Receive a service which time length is a positive random variable with distribution function $G(.)$ and mean α
- When they arrive, each one finds immediately an available server
- Each customer service is independent from the other customers' services and from the arrivals process
- The traffic intensity is $\rho = \lambda\alpha$.

It is easy to understand how the $M\,|\,G\,|\,\infty$ queue can be applied to the unemployment study. Then

- λ is the rate at which occur the firings, supposed to occur according to a Poisson process
- The service time is the time between the worker firing and the moment he/she finds a new job.

In a queue system a busy period is a period that begins when a costumer arrives at the system finding it empty, ends when a costumer abandons the system letting it empty and in it there is always at least one customer present. So in a queuing system there is a sequence of idle and busy periods.

In the $M\,|\,G\,|\,\infty$ queue system the idle periods have an exponential length with mean λ^{-1}.

But the busy period's distribution is much more complicated. In spite of it, it is possible to present some results as it will be seen.

For what interests in this work

- A busy period is a period of unemployment
- An idle period is a period of full employment.

The results that will be presented are on unemployment periods length and their number in a certain time interval.

2. Unemployment Periods Length

Call D the random variable unemployment period length. According to the results known for the $M\,|\,G\,|\,\infty$ queue busy period length distribution

- $$E[D] = \frac{e^{\rho}-1}{\lambda} \qquad (2.1)$$

whichever is a worker unemployment time length distribution (see Takács, 1962)

- As for $Var[D]$ depends on the whole unemployment time length distribution probabilistic structure. But Sathe (see Sathe, 1985) demonstrated that

$$\lambda^{-2}\max\left[e^{2\rho}+e^{\rho}\rho^2\gamma_s^2-2\rho e^{\rho}-1;0\right]\le Var[D]\le\lambda^{-2}\left[2e^{\rho}\left(\gamma_s^2+1\right)\left(e^{\rho}-1-\rho\right)-\left(e^{\rho}-1\right)^2\right] \quad (2.2),$$

where γ_s the unemployment time length coefficient of variation

- If a worker unemployment time length distribution function is

$$G(t)=\frac{e^{-\rho}}{\left(1-e^{-\rho}\right)e^{-\lambda t}+e^{-\rho}}, t\ge 0, \quad (2.3)$$

the *D* distribution function is

$$D(t)=1-\left(1-e^{-\rho}\right)e^{-e^{-\rho}\lambda t}, t\ge 0 \quad (2.4),$$

(see Ferreira, 1991)

- If the unemployment time length of a worker is such that

$$G(t)=1-\frac{1}{1-e^{-\rho}+e^{-\rho+\frac{\lambda}{1-e^{-\rho}}t}}, t\ge 0 \quad (2.5)$$

the D distribution function is

$$D(t)=1-e^{-\left(e^{\rho}-1\right)^{-1}\lambda t}, t\ge 0 \quad (2.6),$$

see (Ferreira, 1995)

- For α and ρ great enough (very intense unemployment conditions) since $G(.)$ is such that for α great enough $G(t)\cong 0, t\ge 0$,

$$D(t)\cong 1-e^{-\lambda e^{-\rho}t}, t\ge 0 \quad (2.7),$$

(see Ramalhoto and Ferreira, 1994).

Note:

- As for this last result, begin noting that many probability distributions fulfill the condition $G(t)\cong 0, t\ge 0$ for α great enough. The exponential distribution is one example.
- As for the meaning of α and ρ great enough, computations presented in Ramalhoto and Ferreira (1994) show that for $\lambda=1$, after $\rho=10$ it is reasonable to admit (2.7) for many distributions.

Calling N_D the mean number of unemployed people in the unemployment period, if $G(.)$ is exponential

$$N_D = e^{\rho} \quad (2.8).$$

For any other $G(.)$ probability distribution

$$N_D \cong \frac{e^{\rho(\gamma_s^2+1)}\left(\rho(\gamma_s^2+1)+1\right)+\rho(\gamma_s^2+1)-1}{2\rho(\gamma_s^2+1)}$$

(2.9),

(see Ferreira and Filipe 2010). Of course, multiplying (2.8) or (2.9), as appropriate, by the mean cost of each unemployment subsidy it is possible to estimate the assistance costs caused by the unemployment period.

Be $p_{1'0}(t)$ the probability that everybody is working at time *t*, being the time origin the unemployment period beginning. Being $h(t)=\frac{g(t)}{1-G(t)}$, where $g(t)$ is the probability density function associated to $G(.)$, the service time hazard rate function[1],

$$h(t)\ge\lambda \Rightarrow p_{1'0}(t) \text{ is non-decreasing} \quad (2.10)$$

,

see Proposition 3.1 in Ferreira and Andrade (2009). And calling $\mu(1',t)$ the mean number of seek people at time *t*, being the time origin the pandemic period beginning

[1]That is: the rate at which unemployed people finds a job.

$h(t) \leq \lambda \Rightarrow$
$\mu(1',t)$ is non − decreasing (2.11)
,

see Proposition 5.1 in Ferreira and Andrade (2009).

3. Mean Number of Unemployment Periods in a Time Interval

After the renewal processes theory, see (Çinlar, 1975), calling $R(t)$ the mean number of unemployment periods that begin in $[0,t]$, being $t=0$ the beginning instant of an unemployment period, it is possible to obtain, see (Ferreira, 1995),

$$R(t)=e^{-\lambda\int_0^t[1-G(v)]dv}+\lambda\int_0^t e^{-\lambda\int_0^u[1-G(v)]dv}du \qquad (3.1)$$

and , consequently,

$$e^{-\rho}(1+\lambda t)\leq R(t)\leq 1+\lambda t \qquad (3.2),$$

see Ferreira (2004).

Also,

A) $G(t)=\dfrac{e^{-\rho}}{(1-e^{-\rho})e^{-\lambda t}+e^{-\rho}}, t\geq 0$

$$R(t)=1+\lambda e^{-\rho}t \qquad (3.3)$$

B) $G(t)=1-\dfrac{1}{1-e^{-\rho}+e^{-\rho+\frac{\lambda}{1-e^{-\rho}}t}}, t\geq 0$

$$R(t)=e^{-\rho}+(1-e^{-\rho})^2+\lambda e^{-\rho}t+e^{-\rho}(1-e^{-\rho})e^{-\frac{\lambda}{1-e^{-\rho}}t} \qquad (3.4)$$

C) $G(t)=\begin{cases}0, t<\alpha\\1, t\geq\alpha\end{cases}$

$$R(t)=\begin{cases}1, t<\alpha\\1+\lambda e^{-\rho}(t-\alpha), t\geq\alpha\end{cases} \qquad (3.5)$$

D) If the unemployment time length is exponentially distributed

$$e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)}+\lambda e^{-\rho}t\leq R(t)\leq e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)}+\lambda t \qquad (3.6)$$

4. Concluding Remarks

So that this model can be applied it is necessary that the firings occur according to a Poisson process at constant rate. It is an hypothesis that must be tested.

Among the results presented, (2.1), (2.2), (2.7) and (3.2) are remarkable for its simplicity and also for requiring only the knowledge of the firings rate λ, the mean unemployment time α, and the unemployment time variance.

The other results are more complex and demand the goodness of fit test for the distributions indicated to the unemployment times.

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