Slack- based Measures of Efficiency in Two-stage Process: An Approach Based on Data Envelopment Analysis with Double Frontiers

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Abstract - **Data envelopment Analysis (DEA) is a mathematical technique for evaluating the relative efficiency of Decision Making Units (DMUs) that convert multiple inputs to multiple outputs. DEA is considered to find optimistic efficient performers in most favorable scenario while giving most favorable weights to inputs and outputs of every DMU. The obtained efficient DMUs construct an optimistic efficient (best-practice) frontier. On the other hand for the purpose of identifying bad performers in most unfavorable scenario, pessimistic DEA model has been proposed, which measures the efficiency with the set of most unfavorable weights. The obtained pessimistic efficient DMUs construct pessimistic (worst-practice) frontier. In many real life situations, DMUs may have a two-stage structure where the first stage uses inputs to produce outputs (called Intermediate) then in second stage that intermediate measures are taken as inputs to produce the final outputs. Assuming this type of structure of production process we used a Slack-based Model (SBM) for obtaining Optimistic and Pessimistic DEA models for stage one, stage two and for overall system in order to measure optimistic and pessimistic efficiencies. An example of non-life insurance industry of Taiwan is selected for supporting our model.**

Keywords: Data Envelopment Analysis (DEA), Slack-based Model (SBM), Two-stage process, DEA with Double Frontier.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric technique to measure relative efficiency and performance of each member of set of related comparable entities, called Decision Making Units (DMUs) and was originally developed by Charnes et al. (1978), assuming constant returns to scale and later extended by Banker et al (1984) to include variable returns to scale. DEA generalizes the single-input single-output case to multiple-input multiple-output case as was given by Farrell (1957). A DMU is considered to be efficient if and only if no other DMU can produce more output by using same inputs or same outputs by using less input. DEA does not require an explicit functional form of inputs and outputs as in parametric methods. But it finds the best DMU and estimates the relative efficiency of other DMUs with respect to efficient DMU. Since DEA can evaluate the relative efficiency of set of DMUs but it cannot find source of inefficiency present in DMUs because conventional DEA views DMUs as black boxes that consume set of inputs to produce set of outputs Avkiran (2009). Using single-stage DEA in such type of cases may result in inaccurate efficiency measurement. Rho (2007) shows that two-stage DEA model allow us to further investigate the structure and process inside the process. In many real life situations DMUs can have a two-stage structure where the first stage produces output by using initial input and that output becomes the input of the second stage to produce final outputs. Output of first stage is equal to input of second stage and is called as intermediate measure as is shown in figure 1.

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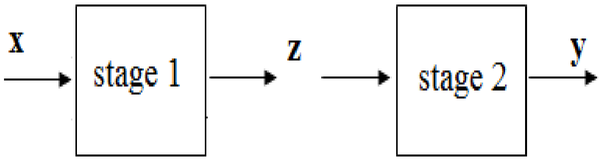

Figure 1. Two stage process for the Jth DMU

Where there are "m" initial inputs used to produce "D" intermediates in first stage. In second stage these "D" intermediate measures are used to produce final "s" outputs.

Wang et al. (1997) used two stage-DEA model to study the impact of IT investment on banking performance. In the first stage of two-stage DEA model banks accumulate funds through deposits and in second stage use these deposits banks invest in securities and provide loans. There are many studies on two-stage production process. For example, Kao and Hwang (2008) developed a model for decomposing overall efficiency into the product of efficiencies of two stages. Chen et al. (2009) also presented a model similar to Kao and Hwang (2008) but is in additive form. Seaford and Zhu (1999) used a two-stage network model to measure the profitability and marketability of American Commercials Banks. Labor and assets was taken as inputs to produce profitability in the first-stage and using profitability from first stage and marketability as inputs in the second stage to produce market value and earnings per share as outputs. Zhu (2000) also uses two-stage DEA model to evaluate the financial efficiency of the best 500 companies. Schinnar et al (1990) used two-stage network to find efficiency of different mental health programs. A baseball performance in two-stage process was given by Sexton and Lewis (2003). Thus two-stage DEA has been used in various dimensions in order to calculate more accurate performance of each DMU. For example, physician care performance by Chilingerian and Sherman (2011), Information Technology efficiency in two stages by Chen and Zhu (2004), Education performance was obtained by Lovell et al (1994).Tone and Tsutsui (2009) argued that one should be careful while measuring the efficiency of DMU through radial DEA models for a two-stage process, because radial efficiency assume that all inputs or outputs change proportionately, so they introduced a Slack-based Measure (SBM)to develop a network DEA approach to evaluate the efficiencies. Later so many modifications came to this model for example Chen et al (2013) showed that this model does not fulfill the property of stage efficiency, suggesting that the rationale for the stage efficiency must be reconsidered. Other extensions have been proposed by Tone and Tsutsui (2010, 2014), Fukuyama and Weber, Kao (2014) etc.

In all the above models we maximize the ratio of weighted sum of outputs to weighted sum of inputs called as best relative efficiency or sometimes called as optimistic relative efficiency or simply optimistic efficiency. In traditional type of models, we solve linear programming problems for each DMU under evaluation and finds a set of optimal favorable weights that maximizes the corresponding optimistic efficiency of each DMU. We call a DMU as optimistic efficient if it's optimistic efficiency is equal to one, otherwise it is said to be optimistic non-efficient. On the other hand, if we minimize the weighted sum of outputs to weighted sum of inputs, the resulting efficiency is called pessimistic efficiency or worst relative efficiency. In this method we get set of most unfavorable weights which minimize the pessimistic efficiency. Thus, we get two frontiers one is optimistic and another is pessimistic frontier. All the DMU will lie between these two frontiers. Figure (2) shows the structure of double frontiers.

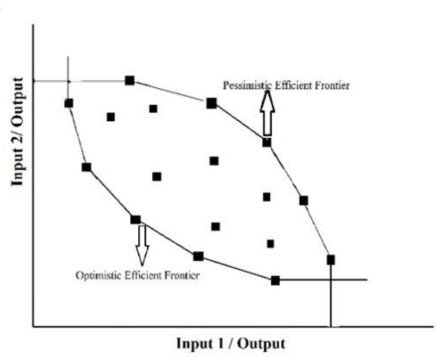

Figure 2. The Structure of Double Frontier

It was Jahanshahloo and Afzalinejad (2006) who ranks the DMUs on the basis of pessimistic efficiency. Azizi and Ajirlu (2011) measure the worst performance of DMUs in the presence of non-discretionary factors and imprecise data. Paradi et al. (2004) uses worst practice DEA in Credit risk evaluation which aims at identifying worst performers by placing them on the frontier. Parkan and Wang (2000) analysis the worst efficiency based on inefficient production frontier. If a DMU has a pessimistic efficiency of one, then it is referred as pessimistic inefficient otherwise, it is called pessimistic non-inefficient. Thus unlike conventional DEA here we have two frontiers optimistic and pessimistic for each DMU, and should be considered simultaneously in analyzing the efficiency in order to give better estimates. For determining the overall performance of each of DMU by considering simultaneously optimistic and pessimistic efficiencies, is said to DEA with double frontier {Wang and Chin (2009), Wang and Chin (2011)}. Azizi et al (2015) used slack-based method for measuring the efficiency with imprecise data by means of double frontier. In fact, the first researchers who measured the overall performance from both perspectives were Entani, Maeda, and Tanaka (2002). In this paper we develop double frontiers in case of two-stage processes with slack-based measure of efficiency.

The remainder of this paper is organized as fallows. In section 2 we have two sections, in section 2.1 we present a general SBM model for measuring the optimistic efficiency of DMUs and in section 2.2 we present SBM model for measuring pessimistic efficiency. Section 3 is followed by presenting SBM models for measuring optimistic efficiency in case of two-stage process. Here we present SBM models for Sub-stages and for overall process for measuring optimistic efficiency. In section 4 we have same models as in section 3 but for measuring pessimistic efficiency. Overall performance measure is measured in section 5.

2. Slack based model for measuring efficiency

A DMU with radial efficiency equal to one and with zero slacks is called CCR efficient. Otherwise, the DMU has disadvantage against the DMUs in its reference set. Therefore, in discussing the total efficiency, it is important to observe both the ratio efficiency and slacks. Some attempts were made to unify radial efficiency and slacks in a single model. Tone (2001) finally formulated the fallowing SBM model for measuring non-radial efficiency.

2.1. SBM model for measuring optimistic efficiency

Suppose that we have n DMUs to be evaluated each consisting of m inputs and s outputs. Let x_{ij} ($i=1,...,m$) and y_{rj} $(\mathrm{r}=1,...\mathrm{s})$ be respectively the inputs and outputs of jth DMU which are known and positive. Then production possibility set is defined as:

$$T=\left\{ (X,Y) \Big/ \sum_{j=1}^{n}\lambda_j X_j \le X; \quad \sum_{j=1}^{n}\lambda_j Y_j \ge Y; \lambda_j \ge 0,\ j=1,...,n \right\} \qquad (1)$$

T is closed and convex set with boundary points as the efficient production frontier. Usual models find the efficiency of DMUs then their slack values, but we can directly assess the efficiency with slack values by the fallowing SBM model given by Tone (2001).

$$\min \rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}$$

$$\begin{aligned} & sub\,to \qquad\qquad (2) \\ & \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io}, \quad i=1,...,\mathrm{m} \\ & \sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{io}, \quad r=1,...,\mathrm{s} \\ & \lambda_j \ge 0; s_i^- \ge 0; s_r^+ \ge 0; \quad j=1,...,n \end{aligned}$$

Where x_{io} and y_{ro} are the inputs and outputs of the DMU under evaluation. $s_i^-(\mathrm{i}=1,..,\mathrm{m})$ and $s_r^+(\mathrm{r}=1,...,\mathrm{s})$ are the input excess and output shortfalls called slacks. The model (2) can be transformed into linear one as fallows;

$$\min \tau = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{io}}$$

$$\begin{aligned} & sub\ to \\ & \mathrm{t}+\frac{1}{s}\sum_{r=1}^{s}\frac{S_r^+}{y_{ro}}=1 \qquad (3) \\ & \sum_{j=1}^{n}\pi_j x_{ij} + S_i^- = tx_{io},\ i=1,...,\mathrm{m} \\ & \sum_{j=1}^{n}\pi_j y_{rj} - S_r^+ = ty_{ro},\ r=1,...,\mathrm{s} \\ & \pi_j \ge 0, (\forall j), S_i^-, S_r^+ \ge 0, \forall (i\, and\, r), \text{and } \mathrm{t} > 0 \end{aligned}$$

Where $S_i^- = ts_i^-$, $S_r^+ = ts_r^+$ and $\pi = t\lambda$ .when for any DMU $\tau^* = 1$ then that DMU is called efficient or optimistic efficient otherwise, it is called optimistic non-inefficient.

2.2. SBM model for measuring pessimistic (worst) efficiency

In the above model we first find the efficient production possibility set and then finding its corner points as optimal efficiency. Now we will find inefficient production possibility set followed by pessimistic efficiency. Azizi and Ajirlu, (2011) defines the inefficient production possibility set as:

$$T = \left\{ \begin{array}{l} (X,Y) \Big/ \sum_{j=1}^{n} \lambda_j X_j \geq X, \\ \sum_{j=1}^{n} \lambda_j Y_j \leq Y, \lambda_j \geq 0, \mathrm{j} = 1,\ldots,\mathrm{n} \end{array} \right\} \quad (4)$$

T is also closed and convex set and its boundary points represent the inefficient production frontier. All the DMUs are compared with the inefficient production frontier. Now we have the SBM in such case as:

$$\begin{aligned} &\max \psi = t + \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^+}{x_{io}} \\ &sub\,to \qquad (5) \\ &t - \frac{1}{s}\sum_{r=1}^{s}\frac{S_r^-}{y_{ro}} = 1 \\ &\sum_{j=1}^{n}\pi_j x_{ij} - S_i^+ = t x_{io}, i = 1,\ldots,m \\ &\sum_{j=1}^{n}\pi_j y_{rj} + S_r^- = t y_{ro}, \mathrm{r} = 1,\ldots,\mathrm{s} \\ &\pi_j, S_i^+, S_r^- \geq 0 \; for\ all\ j,i,r;\ t > 0 \end{aligned}$$

If $\psi^* = 1$ for a DMU then that DMU is called pessimistic inefficient otherwise it is said to be pessimistic non-inefficient. It is usually held that pessimistic inefficient DMUs have worst performance than pessimistic non-inefficient DMUs, whereas optimistic efficient DMU have better performance than optimistic non-efficient DMUs. A pessimistic non-inefficient DMU is not necessarily optimistic efficient.

3. SBM models for measuring Optimistic efficiencies in case of two-stage process

Let us suppose the considered process is of two stage type as shown in fig.1. Suppose there be D intermediates also other than original m inputs and s final outputs as described in section 2. Intermediates are the outputs of first stage and are used as inputs for second stage. Let these intermediate variables for DMU_j are denoted by z_{dj}, $(\mathrm{d} = 1,\ldots,\mathrm{D})$.

As is mentioned above that this type of process cannot be solved by single-stage models thus we will use SBM model to both stages individually, then we will use model for overall efficiency taking into account the operation of sub-processes. The SBM models for estimating the efficiencies with the assumption of constant returns to scale for two stages is given in fallowing two models.

SBM model for measuring optimistic efficiency for stage 1 with $x_{io}(\mathrm{i} = 1,\ldots,\mathrm{m})$ inputs and $z_{do}(\mathrm{d} = 1,\ldots,\mathrm{D})$ outputs for DMU_o

$$\begin{aligned} &\min \tau_1 = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{io}} \\ &sub\,to \qquad (6) \\ &\mathrm{t} + \frac{1}{D}\sum_{r=1}^{D}\frac{S_d^+}{z_{do}} = 1 \\ &\sum_{j=1}^{n}\pi_j x_{ij} + S_i^- = t x_{io}, i = 1,\ldots,m \\ &\sum_{j=1}^{n}\pi_j z_{dj} - S_d^+ = t z_{do}, \mathrm{d} = 1,\ldots,\mathrm{D} \end{aligned}$$

$$\pi_j, S_i^-, \mathrm{S}_d^+ \geq 0, \forall (j, i\ and\ d), \text{and } \mathrm{t} > 0.$$

Similarly, SBM model for stage 2 with $z_{do}(\mathrm{d} = 1,\ldots,\mathrm{D})$ inputs and $y_{ro}(\mathrm{r} = 1,\ldots,\mathrm{s})$ outputs for DMU_o is given as follows:

$$\min \tau_2 = t - \frac{1}{D}\sum_{d=1}^{D}\frac{S_d^-}{z_{do}}$$
$$sub\,to \qquad (7)$$
$$t+\frac{1}{s}\sum_{r=1}^{s}\frac{S_r^+}{y_{ro}}=1$$
$$\sum_{j=1}^{n}\mu_j z_{dj}+S_d^- = tz_{do}, \mathrm{d}=1,...,\mathrm{D}$$
$$\sum_{j=1}^{n}\mu_j y_{rj}-S_r^+ = ty_{ro}, \mathrm{r}=1,...,\mathrm{s}$$
$$\mu_j, S_d^-, S_r^+ \geq 0, \forall (j,d \ \ and\ \ r) \text{ and } t>0$$

Now in order to make the above two models as a single we need to describe the relationship between the two stages. Since the outputs of stage 1 are the inputs of stage 2, so these two quantities must be equal and hence the fallowing constraint guarantees the continuity of two-stages;

$$\sum_{j=1}^{n}\pi_j z_{dj} = \sum_{j=1}^{n}\mu_j z_{dj}, \qquad d=1,...,D \quad (8)$$

Using this constraint, we can develop the SBM model to measure overall efficiency for DMU_o which is given as fallows;

$$\min\ \tau_{overall} = t - \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{io}}$$
$$sub\,to \qquad (9)$$
$$t+\frac{1}{s}\sum_{r=1}^{s}\frac{S_r^+}{y_{ro}}=1$$
$$\sum_{j=1}^{n}\pi_j x_{ij}+S_i^- = tx_{io},\ i=1,...,m$$
$$\sum_{j=1}^{n}\mu_j y_{rj}-S_r^+ = ty_{ro},\ r=1,...,\mathrm{s}$$
$$\sum_{j=1}^{n}\pi_j z_{dj} = \sum_{j=1}^{n}\mu_j z_{dj}, \qquad d=1,...,D$$
$$\lambda,\mu,S^+,S^- \geq 0, \forall (j,i,r\ \ and\ \ d) \text{ and } t>0$$

By using model (9) we can measure the overall efficiency of DMUs by considering the operation of two sub processes. The above model is solved n times for estimating the efficiency of n DMUs. A DMU is said to be optimistic efficient if and only if $\tau_{overall}=1$, otherwise it is said to be optimistic non-efficient. The condition $\tau_{overall}=1$ itself means $s_i = s_r = 0$.

4. SBM models for measuring pessimistic efficiencies in case of two-stage process

Here we will find pessimistic efficiencies of sub-stages as well as for overall stage. For stage 1 the SBM models for measuring the pessimistic are as given as:

$$\max \psi_2 = t + \frac{1}{D}\sum_{d=1}^{D}\frac{S_d^+}{z_{do}}$$
$$sub\,to \qquad (11)$$
$$t-\frac{1}{s}\sum_{r=1}^{s}\frac{S_r^-}{y_{ro}}=1$$
$$\sum_{j=1}^{n}\mu_j z_{dj}-S_d^+ = tz_{do}, \mathrm{d}=1,...,\mathrm{D}$$
$$\sum_{j=1}^{n}\mu_j y_{rj}+S_r^- = ty_{ro}, \mathrm{r}=1,...,\mathrm{s}$$
$$\mu_j, S_d^+, S_r^- \geq 0, \forall (j,d,r) \text{ and } t>0$$

To connect these two sub-processes as a whole process we have fallowing model which measures the pessimistic efficiency of overall process with the assumption that output of stage first is equal to the input of second stage.

$$\max\ \psi_{overall} = t + \frac{1}{m}\sum_{i=1}^{m}\frac{S_i^+}{x_{io}}$$
$$sub\,to \qquad (12)$$
$$t-\frac{1}{s}\sum_{r=1}^{s}\frac{S_r^-}{y_{ro}}=1$$
$$\sum_{j=1}^{n}\pi_j x_{ij}-S_i^+ = tx_{io},\ i=1,...,m$$
$$\sum_{j=1}^{n}\mu_j y_{rj}+S_r^- = ty_{ro},\ r=1,...,\mathrm{s}$$
$$\sum_{j=1}^{n}\pi_j z_{dj} = \sum_{j=1}^{n}\mu_j z_{dj}, \qquad d=1,...,D$$
$$\lambda,\mu,S^+,S^- \geq 0, \forall\, i, \mathrm{d}, \mathrm{r}. \text{ and } t>0$$

In model (12) $\psi_{overall}$ is the overall pessimistic efficiency under most unfavorable conditions for a DMU_o with the assumption of constant returns to scale. When $\psi_{overall}=1$ then DMU_o is called pessimistic inefficient. Otherwise, it is called as pessimistic non-inefficient.

5. Overall performance measure

Here we have two measures of efficiency for each DMU, one is optimistic efficiency measure and another is pessimistic efficiency measure. Thus we need to have an overall efficiency measure for each DMU which considers both the measures. Wang et al (2007) used geometric average of two efficiencies, but we here use another method proposed by Wang and Chin (2009) which is as;

$$\xi_j = \frac{\tau^*_{j(\text{overall})}}{\sqrt{\sum_{i=1}^{n} \tau^{*2}_{i(\text{overall})}}} + \frac{\psi^*_{j(\text{overall})}}{\sqrt{\sum_{i=1}^{n} \psi^{*2}_{i(overall)}}}, \quad j = 1, \dots, n \quad (13)$$

Where $\tau^*_{j(overall)}$ and $\psi^*_{j(overall)}$ are the respectively the optimistic and pessimistic efficiencies of j^{th} DMU. It is clear that overall performance measured by (13) considers magnitude as well as direction of efficiencies, so it is considered to be better than usual geometric average.

Table 1. Selection of non-life insurance companies Kao and Hwang (2008)

Non-life insurance companies	Operation expenses (x1)	Insurance expenses (x2)	Direct written premium (z1)	Reinsurance premium (z2)	Underwriting profit (y1)	Investment profit (y2)
1 Taiwan Fire	1178744	673,512	7,451,757	856,735	984,143	681,687
2 Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3 Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4 China Mariners	601320	594,259	3,174,851	371,863	248,709	177,331
5 Fubon	6627707	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6 Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7 Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8 Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9 Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10 The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11 Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12 Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13 Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14 South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15 Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16 Allianz president	211,716	415,071	5,606,013	402,881	854,054	197,947
17 Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18 AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19 North America	159,422	182,338	1,141,950	483,291	519,121	46,857
20 Federal	145,442	53,518	316,829	131,920	355,624	26,537
21 Royal Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
22 Asia	15,993	10,502	52,063	14,574	82,141	4181
23 AXA	54,693	28,408	245,910	49,864	0.1	18,980
24 Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

6. Numerical Example

In this section, the new approach is applied to the 24 non-life insurance companies of Taiwan as studied by Kao and Hwang (2008). They divided the production process of non-life insurance industry into two stages. Two inputs operational expenses and insurances expenses were used in first stage to produce two intermediates as direct written premium and reinsurance premiums. These two intermediate measures were used as inputs in the second stage to produce two final outputs as underwriting profit and investment profit. The data given in table 1 is directly taken from Kao and Hwang (2008) paper.

In table 2 we obtain optimistic and pessimistic efficiencies of stage 1 in the columns 2 and 3 and some are calculated for stage 2 in the columns 4 and 5. In columns 6 and 7 optimistic and pessimistic efficiencies are obtained for the overall process while considering the effect of intermediate measures also. An overall efficiency measure based on optimistic as well as pessimistic as given by Wang and Chin (2008) is obtained in column 8 under the heading of ξ_j^* .

On the basis of efficiencies obtained in column 8 we can rank the non-life insurance companies with respect their efficiencies. The efficiencies obtained column 8 are on the basis of two-stages as well as on the basis of double frontiers, and is compared with column 12 where the efficiencies are calculated on the basis of double frontier, but the effect of intermediate measures has been excluded. Columns 10 and 12 are respectively the optimistic and pessimistic efficiencies of the process without considering the effect of intermediate measures. In last column we rank non-life insurance companies on the basis of efficiencies calculated in column 12. It can be seen from the table DMUs 12, 15,16,19,24 are optimistic efficient in the first stage where as DMUs 4,10,11,17,20,21,22,24 are bad performers in worst case. In stage two 3, 5,17,20,22 are optimistic efficient. In the overall performance in column 8 DMU 22 has highest efficiency and gets rank one. Similar procedure is done in the column 12 and DMUs are ranked according to their efficiencies where DMU 22 also gets rank one, but this ranking is different to another DMUs.

Table 2. Optimistic & Pessimistic efficiencies in two-stage process

DMU	***Stage 1***		***Stage 2***		***Network process***		***Overall***	***Overall***	***Without considering intermediate Stages***			
DMU (1)	*Optimistic (2)*	*Pessimistic (3)*	*Optimistic (4)*	*Pessimistic (5)*	*Optimistic (6)*	*Pessimistic (7)*	ξ_j^* *(8)*	*Ranking (9)*	*Optimistic (10)*	*Pessimistic (11)*	ξ_j^* *(12)*	*Ranking (13)*
1	0.616588	0.503621	0.475877	0.205191	0.196322	0.224162	0.389703	09	0.7152	0.30545952	0.4342	08
2	0.604917	0.516085	0.343471	0.473737	0.118216	0.254126	0.292578	12	0.4510	0.8834475	0.3089	10
3	0.414598	0.839411	1	0.064586	0.079108	0.21622739	0.288147	14	0.3463	0.24623407	0.3076	11
4	0.344688	1	0.235169	0.228888	0.054884	0.6414687	0.123406	21	0.1302	0.3617779	0.1101	22
5	0.312687	0.761676	1	0.122214	0.297439	0.207849946	0.500099	06	1.0000	0.4139774	0.5854	04
6	0.887171	0.741351	0.41261	0.127578	0.201019	0.307618848	0.337948	10	0.4202	0.3782435	0.3054	12
7	0.252999	0.834902	0.536025	1	0.154303	0.360490518	0.272335	15	0.3357	0.6492632	0.2556	15
8	0.320068	0.894428	0.513245	0.099468	0.173319	0.362662	0.289272	13	0.3357	0.373576	0.2684	14
9	0.414083	0.627195	0.29114	1	0.106747	0.518644	0.188822	20	0.2248	1	0.1668	19
10	0.239946	1	0.655755	0.191898	0.159532	0.282089	0.31296	11	0.4177	0.268057	0.2996	13
11	0.45524	1	0.373516	0.170602	0.019984	1	0.064986	24	0.0392	0.397044	0.0702	23
12	1	0.66365	0.570951	0.136355	0.38815	0.213875299	0.578282	05	1.0000	0.43048	0.5668	06
13	0.343799	0.72231	0.555428	0.080603	0.121393	0.344981	0.247474	17	0.2628	0.16748167	0.2354	17
14	0.261312	0.830522	0.517669	0.23588	0.143607	0.373884	0.257766	16	0.3100	0.168985152	0.2413	16
15	1	0.569781	0.715442	0.04675	0.400572	0.167482	0.649916	03	0.8691	0.26485714	0.6459	03
16	1	0.234455	0.390373	0.475145	0.335336	0.120196	0.698105	02	1.0000	0.479506692	0.6897	02
17	0.184514	1	1	0.363446	0.242534	0.218329	0.438262	08	0.5535	0.203296293	0.4098	09
18	0.603706	0.64337	0.325429	0.16272	0.129042	0.413686	0.232269	18	0.2795	0.186768	0.2170	18
19	1	0.450332	0.431175	0.347461	0.223977	0.172711	0.477087	07	0.6189	0.855688	0.4974	07
20	0.595936	0.503621	1	0.108503	0.352223	0.147372	0.642679	04	0.7542	0.120268	0.5702	05
21	0.523153	0.516085	0.357115	1	0.138559	0.490311	0.223611	19	0.2654	1	0.1604	20
22	0.414778	0.839411	1	1	0.373583	0.109357	0.771962	01	1.0000	1	0.8243	01
23	0.62381	1	6.82E-07	0.36989	3.99E-07	0.710059	0.065315	23	0.0000	0.30545952	0.0562	24
24	1	0.761676	0.160198	0.473737	0.064019	1	0.105991	22	0.1732	0.8834475	0.1181	21

7. Conclusion

Since slack-based measure deals directly with the input excesses and output shortfalls of the DMUs concerned. A DMU with unit efficiency is concerned to be efficient and at the same time all slacks are zero. So in this paper we used slack-based measure in two-stage process for finding the two extreme frontiers optimistic frontier and pessimistic frontier for stage first stage second as well as for overall stage. We obtained an overall measure based on optimistic and pessimistic frontiers simultaneously. We also obtain two types of efficiencies and ranks DMUs in the overall process without taking effect of intermediate measures in order to compare the results, so that we can know whether there is any effect of intermediate measures or not.

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