Heavy-tailed distribution, GARCH models and the silver returns

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Abstract - **After serving as a medium of exchange for the human society, silver is still widely used in our daily life. From the jewellery, electronic and electrical industries as well as medicine, optics, the power industry, automotive industry and many other industries, silver is still playing a very active role. In addition to the industrial usage, silver also serves as an investment tool for many financial institutions. Thus, it is crucial to develop effective quantitative risk management tool for those financial institutions. In this paper, we investigate the conditional heavy tails of daily silver spot returns under the GARCH framework. Our results indicate that that it is important to introduce heavy-tailed distributions to the GARCH framework and the normal reciprocal inverse Gaussian (NRIG) distribution, a newly-developed distribution, has the best empirical performance in capture the daily silver spot returns dynamics.**

Keywords - *normal reciprocal inverse Gaussian; GARCH model; silver spot returns; specious metals; daily data*

1. Introduction

Silver has been regarded as a form of money and store of value for more than 4000 years. It has been considered as the most widely-used precious metal. Silver is the whitest, most malleable and most conductive metal available. It has enjoyed a variety of uses throughout history, most notably as a form of money and jewelry. According to the study by the Silver Institute, the main demand for silver was for industrial applications (40%), jewelry, bullion coins, and exchange-traded products in 2010. While silver is less rare than gold, it has played a significant role and been used as a currency for more than 2700 years. The name of British pound derives originated from the fact that a British pound was once considered to be worth one pound of sterling silver. Over 14 languages use synonymous terms for silver and money. In fact, the U.S. dollar prior to the Civil War was also backed by silver instead of gold, which served as a back of dollar until 1970s.

As a class of investment assets, the price of silver is driven by speculation and supply and demand, like most commodities. The price of silver is notoriously volatile compared to that of other commodities because of lower market liquidity and demand fluctuations between industrial and store of value uses. Thus, risk management of silver price fluctuations becomes even more urgent than that of other commodities. As well-known in the finance literature, the majority of the asset returns exhibit two stylized facts: heavy tails and volatility clustering. In this paper, we reconsider the two stylized facts, but focus on the returns of daily silver spot prices. As in Guo (2017a), we introduce two types of heavy-tailed distribution into the generalized autoregressive conditional heteroscedasticity (GARCH) framework as in Bollerslev (1987), including the Student's t distribution and the normal reciprocal inverse Gaussian (NRIG) distribution. We are interested in the case if the newly-developed NRIG distribution outperforms the Student's t distribution in fitting the returns of daily silver spot prices.

2. Literature Review

There are quite a few studies on GARCH models and returns of the silver market. However, the majority of the studies focused on either the futures market or inter-linkage between the silver market and other markets. Papadamou and Markopoulos (2014) studied the interrelationship between major exchange rate returns (EUR/USD, GBP/USD, and JPY/USD) and precious metal returns (gold and silver) using a vector autoregressive model in a multivariate asymmetric GARCH framework on the intraday frequency. The authors found a unidirectional volatility transmission from the majority of our currencies (EUR/USD, GBP/USD) to precious metals, and the sluggish response of silver volatility to currency volatility shocks permitted implementation of intraday profitable strategies. Khalifa, Miao and Ramchander (2010) estimated four measures of integrated volatility - daily absolute returns, realized volatility, realized bipower volatility, and integrated volatility by Fourier transformation (IVFT) - for gold, silver, and copper futures markets by using high-frequency data for the period 1999 through 2008. Khalifa, et al. found that IVFT could help the realized volatility proxies produce the smallest forecasting errors of futures prices, and increasing the time frequency of estimating integrated volatility does not necessarily improve forecast accuracy. Akgiray, et al. (1991) investigated the time-series properties of gold and silver spot prices, and found both precious metal price series exhibited time dependence and pronounced generalized autoregressive conditional heteroscedastic (GARCH) effects. Abidin, et al. (2013) examined the joint relationship between the percentage price change and the trading volume of silver and platinum futures contracts traded on Commodity Exchange, Inc. (COMEX) using the daily time series which covering a period of ten years, and found that lagged causality in mean running from the price change to trading volume but not for opposite direction under the original AR-GARCH model. Some other studies on markets interdependence and silver futures include Lucey and Tully (2006), Auer (2015), Bentes (2016), Kruse, Tischer and Wittig (2017), Shen, Meng and Meng (2017), and so on.

Here, we want to investigate the conditional heavy tails under the GARCH framework for the silver returns. The heavy-tailed distributions have been quite frequently investigated under into the GARCH framework to account for conditional heavy tails (leptokurtosis). For instance, Bollerslev (1987) firstly considered the Student's t distribution and the GARCH model so that the Student's t distribution could capture conditional heavy tails of a variety of foreign exchange rates and stock price indices returns. Tavares, et al. (2008) incorporated the heavy tails and asymmetric effect on stocks returns volatility into the GARCH framework, and showed the Student's t and the stable Paretian distribution clearly outperform the Gaussian distribution in fitting S&P 500 returns and FTSE returns. Su and Hung (2011) considered a range of stock indices across international stock markets during the period of the U.S. Subprime mortgage crisis, and show that the GARCH model with normal, generalized error distribution (GED) and skewed normal distributions provide accurate VaR estimates.

In this paper, we follow the model framework in Guo (2017a) and are particularly interested in the NRIG distribution, a newly-developed heavy-tailed distribution. The remainder of the paper is organized as follows. In Section 2, we discuss GARCH models and the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Section 5 concludes.

3. The Models

We consider a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t \tag{2.1}$$

$$\sigma_t^{\ 2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\ \ 2} + \beta_1 \sigma_{t-1}^{\ \ 2} \tag{2.2}$$

where the three positive numbers α_0, α_1 and β_1 are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return μ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. The variable e_t is identically and independently distributed (*i.i.d.*). Two types of fat-tailed distributions are considered: the Student's t and the normal reciprocal inverse Gaussian (NRIG) distributions. The density function of the standard Student's t distribution with ν degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^{\ 2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \qquad \nu > 4 \tag{2.3}$$

where ψ_{t-1} denotes the σ-field generated by all the available information up through time *t-1*.

The NRIG is a special class of the widely-used generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda, \mu, \alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2} / \delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2 + (e_t - \mu)^2} / \alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2 - \beta^2})} \exp(\beta(e_t - \mu)) \tag{2.4}$$

where $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \square$ and: $\delta > 0$, $0 \le |\beta| < \alpha$. When $\lambda = \frac{1}{2}$, we have the normalized NRIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha K_0\left(\sqrt{(\alpha^2-1)^2 + \frac{\alpha^2 \varepsilon_t^2}{\sigma_t^2}}\right)}{\pi \sigma_t} \exp(\alpha^2 - 1) \cdot \quad (2.5)$$

4. Data and Summary Statistics

Figure 1 illustrates the daily silver spot prices in the Chicago Mercantile Exchange. Currently, CME is the largest commodity exchange in the world. It merged with the Chicago Board of Trade in July 2007 to become the largest commodity derivative exchange. The data covers the period from June 21, 1991 to June 30, 2017. There are in total 7508 observations. Figure 2 illustrates the dynamics of the silver spot returns. There are significant volatility clustering phenomenon and high volatilities are observed in the Great Recession period.

Figure 1: Daily silver spot prices

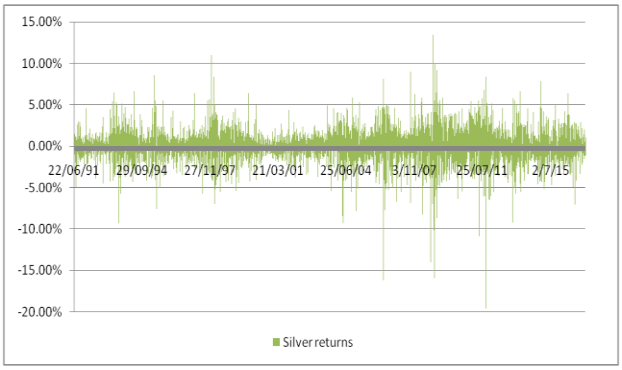

Figure 2: Daily silver spot returns

Table 1 presents summary statistics of the data. The data present the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability and strong evidence of predictability in volatility. All series are presented in daily percentage growth rates/returns. The Bera–Jarque test conclusively rejects normality of raw returns in all series, which confirms our assumption that the model selected should account for the heavy-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is negatively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where T is the number of observations. The data exhibits statistically significant heavy tails.

Series	Obs	Mean	Std.	Skewness	Kurtosis	BJ	Q(5)	$Q^{ARCH}(5)$	$Q^2(5)$
Silver spot returns	7507	0.03%	1.66%	-0.68*	10.36**	101.2**	9.39*	4.37	37.41**

Table 1: Summary statistics. BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, QARCH(5) is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and Q2(5) is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom.
* and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively.

We use the Ljung-Box portmanteau, or Q, statistic with five lags to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The results that no serial correlation is found confirm our assumption of a constant mean return μ in Equation (2.1). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant for all the series.

5. Estimation Results

The GARCH(1,1) model with the Student's t and the NRIG distributions is estimated by maximizing the following log-likelihood function of equation:

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \qquad (4.1)$$

Table 2 reports estimation results of the GARCH(1,1) model with the two types of heavy-tailed distribution for all the daily silver spot return series. All the parameters are significantly different from zero. There results show that it is crucial to introduce heavy-tailed distributions into the GARCH framework and the NRIG distribution has the best in-sample performance. Since the two distributions has the same number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) also indicate the NRIG distribution has best empirical performance.

	alpha1	beta1	1/nu (1/alpha)	log-likelihood	AIC	BIC
Normal	0.046**	0.903**		-13013	26031	26052
Student's t	0.032**	0.917**	0.136**	-12620	25248	25276
NRIG	0.041**	0.931**	0.734**	-12549	25107	25134

Table 2: Estimation of the GARCH model with heavy-tailed innovations
* and ** denote statistical significance at the 5% and 1% level respectively

6. Conclusion

To explain conditional heavy tails of daily silver spot returns, we introduce a newly-developed heavy-tailed distribution, the normal reciprocal inverse Gaussian, under the GARCH framework in fitting the returns series. We compare its empirical performance with that of the most widely-used heavy-tailed distribution, the Student's t distribution, and the benchmark Gaussian distribution. Our results indicate that it is crucial to introduce heavy-tailed distributions to the GARCH framework and moreover the NRIG distribution has the best performance in capture the daily silver spot returns dynamics.

In this paper, we do not decompose the silver spot returns into several components. Guo (2017b, 2017c) showed the spot returns could be decomposed into a long-run factor and a short-run factor for a variety of energy commodities. It would be interesting to investigate if the silver spot returns could be decomposed into several stochastic factors and if the stochastic factors model could be useful for risk management of the silver commodity. In addition, Glosten, Jagannathan and Runkle (1993) introduce asymmetric responses of conditional volatility to negative and positive shocks under the GARCH framework, and it would be interesting to consider such a leverage effect exists in the daily silver spot returns process

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