Conservation laws derived from systemic approach and symmetry

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Abstract— **We prove mathematically for the first time that a conserved value must exist in economics. We derive it by using symmetry arguments in the systemic approach. Recently, it has been shown that a conserved value and its rigorous application can avoid any form of financial crises. So it is proven that a financial crisis is for sure avoidable without any other economic sacrifices.**

***Keywords*—** *systemic approach; conserved quantities; conserved value; chaos; financial crisis; marketing mix*

1. Introduction

Due to chaos the weather is not predictable at least for longer periods of time like a month. This has been known since the 1960s (see e.g. Schuster (1984)). The same may be true for planning and forecasting in business and economics. It has been noted by Grabinski (2004). The only way to deal with chaos is to use conserved quantities. This is the reason why physicists use conserved quantities like energy to describe otherwise chaotic systems. Later Grabinski (2007) also stated that this might be essential for the financial world. Appel (2011, 2012) has defined a *conserved value* which allows to describe the (mathematically) chaotic financial markets. If applied rigorously the financial crises like the one of 2008 vanishes without a trace. It also helps to explain the otherwise incomprehensible momentum effect (Appel (2012a)). Schädler (2015) showed that a sufficient amount of *speculative money* (= non-conserved value) will *always* lead to economic instabilities, which makes financial crashes inevitable. Klinkova (2017) showed the same for the micro economic world. Financial speculations are therefore not only close to gambling. They are *identical* to gambling.

In this sense financial crises are understood. How to deal with conserved quantities in order to avoid instabilities in *daily life* is another question. Dziergwa (2015) showed that conserved value based accounting principles (CVBAP) are possible but at least tedious. Avoiding speculation by e.g. introducing a Tobin tax is considered a practical approach as noted by Dziergwa (2013).

However, Appel (2011) defined the conserved value of the financial world in a heuristic way. Up to now there is no rigorous proof that a conserved value exists and how to define it in a mathematical way. Exactly this is the purpose of the present publication.

In section 2 we are using a rigorous mathematical approach to derive the conserved quantity of business and economics. It is as fundamental as the derivation of the conserved quantities in classical mechanics (energy, momentum, and angular momentum) which is over 100 years old. We have to admit that we *borrowed* this approach from physics.

Though this derivation is as fundamental as the introduction of conservation laws in physics it lacks its practical impact like e.g. the concept of energy in physics. This comes clear from the critical discussion in section 3. There we show the obstacles from a purely mathematical point of view. In section 4 and 5 we show the shortcomings in practical examples rather than theoretically like in section 3. In section 6 we conclude and summarize our findings.

2. Deriving the conserved quantity

Erich Gutenberg (1897 – 1984) stated more than 50 years ago that there *exists* a function governing business and economics. The essential idea can be found in a reprint (Gutenberg (1998)). Arguably Gutenberg is the father of management science. It is the same approach as in thermodynamics where one demands that there exists a function governing all thermodynamics. It is called entropy there. This analogy did not come by chance. Gutenberg started to study physics and chemistry and turned to economics eventually.

Unlike the entropy, from Gutenberg's fundamental function we do neither know how it looks like nor what are its variables. In most management science models one assumes suitable variables and determines the function either empirically or by a microscopic ansatz and a solution of a differential equation.

Here we assume just the existence of a function L governing the economic world completely or partly. Its variables are kept general as q_i, $\dot{q}_i$, and the time t. The dot refers to the derivative with respect to time. i runs from 1 to n with n being any number. We do not take any higher derivatives. This is correct as long as one assumes arbitrarily fast changing variables but not strict discontinuities. The function L may express the wellbeing of the human society, either in a monetary sense or in another way such as *being happy*. Please note that this assumption is not essential. Because L is just an arbitrary function describing the world in a unique way, one may demand anything of L such as being maximal, minimal, equal to zero … Whether one gets some inside from this assumption is another question. In order to get a useful result we assume that the functional W becomes maximal (maximal wellbeing in the interpretation above):

$$W = \int_{t_1}^{t_2} dt\, L(q_1(t),..,q_i(t),\ldots q_N(t);\ \dot{q}_1(t),\ldots\ \dot{q}_i(t),\ldots \dot{q}_N(t);t)\ \rightarrow\ maximal \qquad (1)$$

Eq. (1) is a typical problem of functional analysis. Dealing with it is well known for over 100 years. Almost any mathematical handbook such as e.g. Bronshtein (2007) shows the way how to prove that Eq. (1) is *identical* to the following set of differential equations:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \qquad (2)$$

Eq. (2) may be considered the equation of motion for business and economics. Though Eq. (2) is rigorous and valid under any circumstances, it is far from being useful. This is because the q_i are not known at all. However, in a deterministic world L must not depend on time explicitly. Else the outcome at a later time would differ from the previous outcome even if *everything* else is identical. This is also known as homogeneity in time. Translated into mathematics we have

$$\frac{dL}{dt} \equiv \frac{\partial L}{\partial q_i}\cdot \dot{q}_i + \frac{\partial L}{\partial \dot{q}_i}\cdot \ddot{q}_i + \frac{\partial L}{\partial t} = \frac{\partial L}{\partial q_i}\cdot \dot{q}_i + \frac{\partial L}{\partial \dot{q}_i}\cdot \ddot{q}_i$$

With the help of the latter equation we find after some rearrangement of Eq. (2):

$$\frac{d}{dt}\left(L - \sum_{i=1}^{N} \dot{q}_i \cdot \frac{\partial L}{\partial \dot{q}_i}\right) = 0 \qquad (3a)$$

In other words

$$L - \sum_{i=1}^{N} \dot{q}_i \cdot \frac{\partial L}{\partial \dot{q}_i} \quad is\ conserved \qquad (3b)$$

Eq. (3a) shows the conserved quantity. Please note that deriving a conserved quantity from symmetry arguments such as homogeneity in time is far from being new. It has been used in physics for over hundred years. Even in newer economic textbook such as Sato (1990) the procedure has been used. Of course, it has never been used to calculate the fundamental conserved quantity named conserved value by Appel (2011).

By deriving Eq. (3a) the main goal of this publication has been achieved. Though not rigorous one may find an interpretation for Eq. (3a) closer to the definition of Appel (2011).

Adam Smith (1723 – 1790) said essentially that everything has a value (in a monetary sense) and maximizing all these values is the goal of mankind. Though modern behavioral economics has proven that this is at least not always true, we stick to this assumption here. Rather than chimpanzee human beings are not homines oeconomici. But if they were, our function L could be expressed as a function of values v_i and there changes in time $\dot{v}_i$. A lowest order Taylor expansion of L yields

$$L = L_0 + a_i \cdot v_i + b_i \cdot \dot{v}_i + O\left(v_i{}^2,\ \dot{v}_j{}^2, v_i \cdot \dot{v}_j\right) \quad (4)$$

Here Einstein's sum convention has been applied. Chopping the world into smaller and smaller pieces makes the v_i arbitrarily small (while n increases to infinity). The same is true for the $\dot{v}_i$ if one takes a smaller and smaller time scale. In this limit the lowest order Taylor expansion of Eq. (4) becomes exact. So we can insert the lowest order of Eq. (4) into expression (3b) and conclude

$$L_0 + \sum_{i=1}^{N} a_i \cdot v_i \quad is\ conserved \qquad (5)$$

Because L_0 is a constant it can be dropped in expression (5). So a linear combination of all values must be conserved. This is the line of argumentation Appel (2011) took.

3. Critical discussion

At first glance the derivation of section 2 has no limitations. Though often ignored, there is a variety of prerequisites for its validity.

First of all we started with an optimization problem. Obviously all variables must be an element of an ordered field (in an algebraic sense). Though it sounds trivial, it is not always the case even in physics. In quantum mechanics the fundamental variable is complex valued. (Normally called wave function ψ) And the field of complex numbers is not ordered. Therefore the otherwise very general approach of classical mechanics does not work for quantum mechanics.

In business and economics the variables must be expressible as a (real) number. If they are, they are for sure ordered. The variable price does fit in this category. The variable *placement* (cf. section 4) has no natural value (number) assigned. Even if one assigns an (artificial) number it is rarely the case that these numbers will build a field. It is even quite difficult to define an addition and multiplication which does make any sense.

Another limitation has been mentioned in section 2 already. We took the first order derivative of each variable only. Knowing the present value of a variable and how it is changing in time determines all future values. This is true as long as there are no discontinuities in the variables. This will not cause any problems in business and economics. Even if there were a discontinuity, it can always be approximated by a sufficiently fast changing function. This is the same line of argumentation one would apply in classical mechanics. Please note that in quantum mechanics the world is different. There we have a measurement process which changes the wave function strictly discontinuous. Therefore any derivative of the wave function must be taken into account.

The last two prerequisites are a generalization of the latter one. If one would take into account higher derivatives of time, they would not be independent of the zeroth and first order derivative. On the other hand not taking the first order derivative would make the set of variables incomplete.

As a generalization, our variables must be complete and independent. Complete means that knowing all the values of each variable determines the system uniquely. In other words, if the variables of two systems show pairwise the same value these systems are *identical*. Independent means that the value of no variable can be determined by the values of (any part of) the other variables.

Demanding completeness and independence of all variables in business and economics has been stated by Grabinski (2004) already. It is the greatest obstacle in deriving a "world formula" for business and economics. The next two sections will give some examples for it.

4. Example Marketing Mix

Probable every business major knows the term marketing mix, and he or she can recite the four Ps: product, promotion, price, and placement. It is described in most textbooks such as Kotler (2016). For services one normally takes two additional Ps: people and process.

To see how a product or service will behave in the market one takes proper Ps and "discusses" how they interact with each other. In most cases this is done in words rather than numbers. But there are examples of a strict quantitative analysis such as Selvarasu (2017).

Surprisingly, the background of the marketing mix is less well known. It is sometimes also called *McCarthy approximation* after Edmund Jerome McCarthy (1928 – 2015). Approximation means that the fundamental function of marketing (our L of section 2) is assumed to have only four variables, the four Ps. That the variables of the marketing mix always start with a P is nothing but marketing for the marketing mix. Choosing meaningful Ps for a particular setup is one of the essential ingredients in using the marketing mix.

Once having properly defined Ps and assigning values (numbers) for it (cf. e.g. Selvarasu (2017)), it is *formally* simple to find an *equation of motion* for marketing from Eq. (3a):

$$L = \dot{q}_1 \cdot \frac{\partial L}{\partial \dot{q}_1} + \dot{q}_2 \cdot \frac{\partial L}{\partial \dot{q}_2} + \dot{q}_3 \cdot \frac{\partial L}{\partial \dot{q}_3} + \dot{q}_4 \cdot \frac{\partial L}{\partial \dot{q}_4} \quad (6)$$

In this example one may take $\mathrm{q_1 \equiv price}$, $\mathrm{q_2 \equiv product}$, $\mathrm{q_3 \equiv promotion}$, and $\mathrm{q_4 \equiv placement}$. Eq. (6) is a (simple?) differential equation. Together with Eq. (2) it determines the marketing world once and forever, or doesn't it?

Unfortunately (or fortunately if you are a marketing expert) it is too good to be true. Almost all our requirements of section 3 are violated. Therefore Eq. (6) is in general invalid. The $\dot{\mathrm{q}}_\mathrm{i}$ may have numbers (or values) like in Selvarasu (2017). However, the $\dot{\mathrm{q}}_\mathrm{i}$ do not form an algebraic field. Furthermore they are far from being independent. E.g. the price cannot vary independently from the product (and especially its quality). As long as the marketing mix is properly defined, $\dot{\mathrm{q}}_\mathrm{i}$ may form a complete set of variables at least in good approximation. However, even this is doubtful because the chosen McCarthy approximation origins most likely not from the demand of *mathematical* completeness.

5. Example of Hofstede's cultural dimensions

While discussing different cultures, people normally do not expect mathematics. Though empirically, Geert Hofstede did just that. He assigned scores for different traits in different cultures. The details can be found in many textbooks such as Hofstede (1991).

The general approach goes as follows. The culture of a particular country is defined by six variables:

$$q_{PD} \equiv \textit{Power Distance} \quad (7a)$$

$$q_{Id} \equiv \textit{Individualism} \quad (7b)$$

$$q_{Mc} \equiv \textit{Masculinity} \quad (7c)$$

$$q_{UA} \equiv \textit{Uncertainty Avoidance} \quad (7d)$$

$$q_{LO} \equiv \textit{Long Term Orientation} \quad (7e)$$

$$q_{Ig} \equiv \textit{Indulgence} \quad (7f)$$

Though it is not essential here, just a few words about the meaning of these variables:

Power Distance	degree of acceptance of unequally distributed power
Individualism	degree of "I" instead of "we"
Masculinity	degree of how much people are motivated by being the best
Uncertainty Avoidance	degree how much people avoid (being threatened) by uncertainty
Long Term Orientation	degree how much people are orientated by the past to solve problems
Indulgence	degree of how much people control their impulses or desires

For each trait a score of zero to hundred is assigned. Just consider the three countries Germany, Russia, and USA. Their score for e.g. individualism is 67, 39, and 91, respectively. So Germany (67) is pretty individual, Russia (39) is almost collectivist, and the USA (91) is extremely individual. (Please note that it is not forbidden to put the six numbers of a country in an n-tuple, e.g. for Germany one has the numbers (35, 67, 66, 65, 83, 40). However, these six numbers are not a six dimensional vector, because they are not building a vector space. Such a mistake can be found quite often (especially in economics).

Assuming that the scores for each country describe the average value for each trait rather than the trait of a typical person of the particular country, these variables build an ordered field. (They are intrinsic variables just like the temperature in thermodynamics) So it seems justified to use Eq. (3a) in order to write down the equivalent equation to Eq. (6). That is the main reason why we have chosen this example. If true one could predict possible changes in culture over time. In particular Eq. (3a) would predict what happens to the other scores if one score goes down. Again this is too good to be true.

Though Hofstede's cultural dimensions look like perfect candidates for our variables, they are neither complete nor (most likely) independent. The lack of completeness is easily proven. If two countries would show the same six numbers, these countries must be (culturally) identical. Or in other words the culture of a society is determined by six numbers only. Then all the books about cultures were just an assembly of redundancies. Luckily studying cultures is not such a boring enterprise.

Disproving independence is hardly possible. But it is not plausible that all cultural dimensions are *completely* independent. Of course, the cultural dimension of Hofstede have been chosen carefully which makes them valuable. So they are probably only slightly dependent. But be it as it may, an *equation of motion* for the cultures is not possible to state, at least not from Hofstede's approach.

Lacking completeness is a problem for many other reasons. The political system (at least if not forced upon the citizens) is a mirror of the country's culture. Therefore it is very surprising that people speak of *left* or *right* only, if they are talking about politics. That would make (political) culture one-dimensional. (The expression right and left has not been invented by political science. It just denotes who set on which side in the Paulskirche in 1848) This explains why there are so many different leftwing or rightwing directions. Or sometimes politicians from the right and left can form a coalition. They have probably much in common in other cultural dimensions. Political scientists therefore sometimes speak of four basic political systems: communism, socialism, liberalism, and fascism. It is still short of two dimensions if compared to Hofstede.

6. Conclusions

We have shown that a conserved quantity does exist. The conserved value of Appel (2011) is therefore not just a hand waving tool. It is the ultimate proof that financial crises and the likes are avoidable if one sticks to a properly defined value instead of mixing up value with price.

There are quite a few mathematical restrictions in order to use our approach. (Variables must be complete, independent, and elements of an ordered field; there must not be strict discontinuities) This is the reason our examples (marketing mix, cultural dimensions) do not yield surprising results.

Please note that all the restriction (cf. section 3) are very useful to be considered in almost all quantitative analysis when mathematics is used to make conclusions.

Our goal was not to find a useful tool. It was to prove the principle. If the restrictions in section 3 were not possible to fulfill for principle reasons, then any kind of quantitative analysis is not possible.

As a challenge for the future one should try to find other "conserved quantities" in business and economics. As seen in sections 4 and 5, it can be tricky.

As mentioned in section 2, this symmetry approach has been used in economics already, cf. Sato (1990). However, the rigorous prerequisites stated in section 3 where neither mentioned nor proven. This is especially important in theoretical considerations. Paul A. Samuelson (1915 – 2009) stated *the law of conservation of income growth* and *the law of the capital output ratio*. (For details see e.g. Sato (1990), p. 57–70) However, he never considered whether his variables were complete. It remains to be proven if this is the reason why reality sometimes deviates from his theory.

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