International Journal of Finance, Insurance and Risk Management
Volume XII, Issue 1, 2022

pp. 49-79

Bitcoin Volatility Estimate Applying Rogers and Satchell Range Model*

Submitted 12/11/21, 1st revision 13/12/21, 2nd revision 12/01/22, accepted 15/02/22

Eloisa Cadenas-Morales[1], Edgar Ortiz[2]

Abstract:

Purpose: *The objectives of this paper are, first, to synthesize the main events that have influenced the development and volatility of bitcoin, and second to present and apply the Rogers and Satchell range model (1993) for the measurement of bitcoin volatility.*
Desigh/methodology/Approach:
Findings: *The results are useful for risk management, searching profitable investments, diversification of portfolios, and the application of a reliable risk parameter for the valuation of bitcoin financial and real options. The evidence suggests further studies extending it, considering leptokurtosis and other moments of the distribution of the series.*
Practical implications: *Finally, the study highlights the need for clear regulations on bitcoin and other cryptocurrencies to ensure a fruitful future co-existence with digital currencies created by the central banks. The work suggests the creation of additional IMF's Special Drawing Rights in lieu of a global cryptocurrency. That would help to overcome the problems created by the Covid-19 pandemic and member countries would retain their monetary sovereignty.*
Originality value: *These objectives underline the originality and contribution of the work. This is the first time this model is fully recognized and used, highlighting its advantages.*

Keywords: *Bitcoin volatility, Rogers and Satchell, range models, cryptocurrencies, central bank digital currencies.*

JEL Classification: *C01, C13, C58, D53, F37*

Paper type: *Research article.*

[1]*Universidad Nacional Autónoma de México*
[2]*Universidad Nacional Autónoma de México, edgaro@unam.mx;*
**Paper presented in ICABE 2022 www.icabe.gr*

1. Introduction

The daring innovations of bitcoin and ulterior altcurrencies –decentralized, non-fiat currencies based on blockchain technique– came immediately to the attention of investors, regulation authorities, entrepreneurs, consumers, and the academia leading to controverted points of view and research. Their detractors stressed their lack of legality, scalability vs. other forms of digital payments (mainly credit cards), unknown security against hackers, misuse to hide illicit operations like money laundering, excessive volatility, lack of inherent value, and the lack of regulations.

Their defenders welcomed their appearance even though their original characteristics were limited and unknown. They stressed the possibility of peer-to-peer faster transactions, smaller transaction fees, greater confidentiality of transactions, easing of international trade, individual control of one's account, and the potential alternative for new and greater investment and credit opportunities.

Strong debates took place particularly in the professional and academic worlds. Due to its initial reduced acceptance and limited knowledge about their market behavior bitcoin and other cryptocurrencies were mostly considered as volatile speculative investments, far from becoming fully currencies assets accepted worldwide for local and international transactions and as mean of exchange.

Indeed, the road full of favorable and unfavorable situations paved the way to these currencies' volatility. Considering their importance of this intricate path of development of bitcoin, the first objective and contribution of this paper is to highlight some facts related to the erratic behavior of bitcoin's prices and returns. The second objective and contribution of this work is to present and apply a novel method to measure the volatility of financial assets, Rogers and Satchel range model (1993). This methodology has been practically ignored to measure the volatility of bitcoin.

2. Key Developments in Bitcoin's Acceptance

Bitcoin and some cryptocurrencies have shown increased trade and favorable developments since the last decade. A major impetus for bitcoin trading and acceptance was induced by listing of bitcoin futures and options in the CME, the CBOE, and NASDAQ. Additionally, many start-up projects began to be financed by Initial Currency Offerings (ICOs), cryptocurrency offerings equivalent to initial public offerings (IPOs), launched by a company to gather financing for specific projects, overcoming conventional regulated fundraising processes (Sosa *et al*., 2020).

A contemporaneous innovation to overcome high volatility of cryptocurrencies was the creation of stable cryptocurrencies by some crypto exchanges, aiming at enhancing their stability and promoting greater acceptance and trade. Essentially, stable cryptocurrencies constitute either a blend between fiat money and cryptocurrency, or else a cryptocurrency pegged to a hard currency, or else some other asset like oil or gold. In despite of its legal problems, Tether, pegged to the dollar, is the most important stable cryptocurrency; about 80% of all bitcoin trading is done with the help of Tether, which secures liquidity of the crypto market.

However, despite of the proliferation of crypto currencies, bitcoin is the dominant crypto since the appearance of these currencies. By January 2021 Bitcoin performance started the year with a strong performance. Bitcoin's price started at $31,971.91 and its market dominance amounted to 62.9% among a total of 8,361 cryptocurrencies and evolved favorably reaching an all-time high price of $64,800 by April 2021. However, bitcoin volatility increased sharply, and its price registered a downfall of over 30% after the China Banking Association announced last May that it will not continue using cryptocurrencies. Moreover, its dominance, although is still the most important has declined to 45.91% (now among now 10,747 cryptocurrencies) and its 24-hour trading volume has declined to $23,597,077,704 from $70,746,624,882 from January 1, 2021 (CoinMarket, 2021)

At any rate, three other important facts complete the inroad of bitcoin and other altcoins to their acceptance and legitimization in the financial markets. First, a clear tendency of governments and their regulation authorities to accept cryptocurrencies trade and payments, namely in stable cryptocurrencies; this event in underlined by the decision of the United States Office the Controller of the Currency to let regulated banks to carry payments in stable cryptocurrencies; moreover, banks can now make use of public block chains to store, process, validate, record, and make payment transactions (Office of the Comptroller of the Currency, 2020).

A more progressive position is that of Japan. Its regulation recognizes Bitcoin and other digital currencies, now denominated “crypto-assets” in lieu of “virtual currencies”, as legal property under the Payment Services Act (PSA). Since the 2020 regulations, crypto exchanges are required to be registered and comply with traditional Anti-Money Laundering/Combating Financing of Terrorism (AML/CFT) controls Finally, beginning April 2021 Japan began experimenting on issuing its own cryptocurrency (ComplyAdvantage, 2021).

Second, since last August 2020, important corporations among them MicroStrategy, Stone Ridge Asset Management, Model Global Investment, and Grayscale began expanding their portfolios and investments with large purchases of bitcoin. Additionally, recently TELSA, Mastercard, and PayPal have fostered bitcoin trading. Indeed, last February 8, TELSA purchased $1.5 Billion USD (crypto pegged to the U.S. dollar) and soon will accept payments in dollars. The Securities Exchange Commission has also come to the scene. Due to the still restricting or unclear regulations regarding digital currencies, the SEC holds several lawsuits against several cryptocurrencies and their executives (Ripple, Ethereum, Litecoin) because ignoring to register ICO operations.

Still, more important are the continuous growth and acceptance of diverse offerings of digital cryptocurrencies beyond its original role of a store of value. In addition, to the advances previously pointed out convertible notes are now issued in bitcoins. Microedge has issued 1.05 billion of convertible note due in 2027. The notes are convertible into cash, shares of MicroStrategy's class A common stock, or a combination of cash and shares of MicroStrategy's class A common stock, at MicroStrategy's election (Microstrategy, 2021).

Third, induced by the bitcoin bull market, Coinbase Global Inc., the largest U.S. cryptocurrency, is listed in the NASDAQ stock market since April 14, 2021. It is also trading at the Frankfurt Stock Exchange. Coinbase declares over $20 billion in trading volume and over 10 million registered users; it offers access to bitcoin, bitcoin cash, Ethereum, and Litecoin. Coin. shares listed reference price $240.00 undertook at once an explosive growth followed by most tempered trends, but characterized by sharp volatility, following in turn the volatility of bitcoin prices. (Yahoo Finance, 2021; Google Finance, 2021).

The increased acceptance of bitcoin, and digital currencies in general, continues at a rapid course. Just one week after its listing, NASDAQ started trading successfully monthly Coinbase options; all other U.S. options markets followed which includes those managed by CBOE Global Markets CBOE.Z, the Nasdaq NDAQ.O and the Intercontinental Exchange ICE.N. About 12,000 Coinbase options contracts changed hands in the first two hours of trading, with calls outnumbering puts slightly. (NASDAQ, 2021).

Nonetheless, not everything looks encouraging for cryptocurrencies. New challenges are emerging, particularly, it is important to note that central banks around the world contemplate to create their own digital currency**.** Likewise, the Chinese banking Association has announced that it will not use digital currencies anymore. In addition, China is expanding its pilot program for a local electronic payment system.

China's commercial hub Shanghai, six big state banks are quietly promoting a digital yuan enforcing a political mandate to provide consumers with a payment alternative to Alipay and WeChat Pay (Reuters, 2021a). Similarly, The Bank of Japan's Digital Currency Committee held recently its first meeting, while China´s Central Bank proposed global rules for central bank digital currencies (CBDCs) last March 25 at the Bank for Institutional Settlements conference.

The Bank of Japan is also working with the European Central Bank, the Federal Reserve, and the Bank of England to research into how CBDCs will work. Similarly, the Federal Reserve, although has continuously mentioned that it does not have plans to launch a digital currency, lately confirmed that it plans to "build and test a hypothetical design." However, its Governor Jerome Powell has indicated that a "great deal of work" needs still to be done so that the central bank would decide to launch a digital dollar (Wall Street Journal, 2020; Market Insider, 2021).

The Treasury and the Bank of England has also created the Central Bank Digital Currency Taskforce which must coordinate the exploration of a potential United Kingdom cryptocurrency denominated in Sterling Pounds (Bank of England, 2021).

Some analysts believe that CBDCs could imperil cryptocurrencies development. Nonetheless, in a current report the European Central Bank rather estimates that cryptocurrencies are speculative assets and not actual currencies. Most likely, CBDCs will still represent the fiat currency that they are replacing, which will give central banks a huge increase in control over monetary policy. However, coexistence of CBDCs, physical cash, true cryptocurrencies like bitcoin, and blockchain-based stablecoins, could take place, each serving different investors preferences (Crypto Daily, 2021; fast, 2021).

Moreover, coexistence most likely will be marked by the additional acceptance of some countries of bitcoin as legal tender. The National Congress of El Salvador approved a law which became enforced last Monday, September 7. This surprising decision is seen as an alternative medium of exchange since El Salvador became a dollarized economy since 2001and the Fed is the institution with full control of the dollar. Nonetheless, El Salvador's decision has been also criticized due to its long run impact of the national debt denominated in dollars (Wall Street Journal, 2021). Indeed, bitcoin crashed the initial day as a legal country causing a loss of US$3.0 million. Moreover, El Salvador bond spreads relative to the U.S. Treasuries reached a maximum the following Thursday due to growing investors' fears that the country will not reach a potential one billion loan agreement with the IMF; those spreads rose to 986 basis points, higher that a record high spread in May 2020 during de initial peak of pandemic crisis (Reuters, 2021b).

In conclusion, a new financial architecture is in the making, but more than just co-existing, profound reforms to the international financial governance is needed. The creation of fully accessible interest bearing CDCDs would have to be coordinated, with rules that homogenize their emission and uses. Costs and the impact CBDCs on interest rates, financial stability, and security must be weighted. To promote international stability, Ocampo, (2021) suggests the creation of an international cryptocurrency, administered by the International Monetary Fund, to be used in private transactions. The IMF could increase and convert special drawing rights (SDRs) into a digital currency.

Although this strategy is appealing negotiations would be very complex and lengthy. Urgent solutions should be undertaken to foster economic and social recuperation after the Covid-19 pandemic worldwide ill effects. SDRs should be incremented along with redistribution of IMF quotas, promoting a greater participation of developing nations. Thus, a reinvigorated international governance would foster increased trade, investments, and growth. All member countries would retain their control of local monetary policies. Moreover, CBDCs would aid member countries to enforce countercyclical policies.

Under the above complex scenario of increasing but still incomplete acceptance of digital currencies, research has contributed significantly to the understanding of their characteristics and potential uses. Many earlier works based their studies on well-known financial markets issues, theories and models about investments and asset management: risk and return, market efficiency, correlation of digitals among themselves, or else with established assets, construction of portfolios combining various cryptos, or else the addition of cryptos to conventional portfolios.

In time, increasing swings in the prices of bitcoin brought the attention of financial economists toward modeling its volatility. However, a good number of studies rather emphasized the quality and complexity of models employed neglecting to link their evidence with financial and real options pricing. All financial pricing models need volatility estimates to assess potential losses or gains. Concerning bitcoin, valuation of its derivatives is very important. Moreover, aside from the futures and options on bitcoin futures, derivatives of other cryptos are already traded at specialized cryptocurrency exchanges. However, research about these issues is limited.

Deribit, based in the Netherlands derivatives exchange is the largest cryptocurrencies derivative exchange. It offers bitcoin futures and European bitcoin options enabling traders to trade them with up to 100 times leverage (Thelleman, 2020). Other bitcoin derivative exchanges are FITX, LedgerX, IQ Option, and Quedex. LedgerX is the only US regulated Bitcoin options exchange and offers physically settled BTC derivatives and it also offers binary bitcoin options (Phillips, 2020). Also, as previously pointed out Coinbase options are now offered by the most important options markets. In all these cases, the volatility parameter is mandatory to value them correctly. Here, a point to underscore is the Black and Scholes (1973) model and its extensions which need the volatility parameter of the underlying asset for its valuation applications. Furthermore, as already pointed out, Coinbase options are now traded in several options exchanges.

Similarly, all valuation models of real assets require of volatility parameters to ensure the feasibility of a project, with particular importance for valuations employing the Black and Scholes real options extensions; corporate investment strategies now include the possibility of both funding their projects with bitcoin, as well as transacting and receiving corresponding investment flows in bitcoin, which can be extensive to other cryptocurrencies. Key projects from all sectors of the economy are expected to carry out projects both financed and generating flows with cryptocurrencies, particularly bitcoin. This is an ongoing development beyond ICOs and tokenized securities.

In his respect, two issues are relevant. First, cryptocurrencies have been identified as environment unfriendly due to their high consumption of electricity used for mining. That turned the environmentally conscious people against cryptocurrencies. To overcome this problem Bitcoin Green is an environmentally friendly finance alternative. It focuses on preventing needless (electricity) waste and environmental harm. It was created by the Bitcoin Green Foundation (BITG), a nonprofit organization committed to developing and maintaining BITG awareness about BITG and cryptocurrencies, as well as educating people about digital currencies and how they impact the environment. Indeed, Co2Bit is the first cryptocurrency created to promote climate change, whereas Peercoin seeks to decrease the high bitcoin mining costs. (Plublius, 2018; Bitgreen, 2021).

Second, besides clean financing, many corporations are interested in implementing real investments environmentally friendly using bitcoin which they already see it as sound money. Moreover, it is already seen as the best defense against the disruption facing the finance industry and central banking, and the core of a new "monetary - architecture" (Taraldsen, 2021). Clean energy projects such as solar energy generation, or else free emissions transportation vehicles are the most favored projects. Important strategic decisions will be involved, and risks will have to be fully assessed.

In this respect, this work benefits of recent literature about an alternative volatility measure to assess the multiple problems associated with bitcoin volatility: the Rogers and Satchell (R&S) range model which incorporates the opening, closing, maximum and minimum prices, and it is applied it in this work.

It is worth noting that Rogers & Satchell model is an unbiased estimator regardless of drift and allows the error not to be affected by the high and low values of the series. In addition to be a solid alternative to estimate volatility, its data requirements are not as complex and expensive to compile as is the case of intraday day data of close intervals. Additionally, according to our revisit to the extant literature, this is the first time it is used to measure the volatility of bitcoin. Fiszeder *et al*. (2019), Tan *et al.* (2020) and Molnár (2021) apply related range methodologies, concentrating in modeling testings, but Rogers and Satchell model is not included.

3. Literature Review

Bitcoin volatility has been examined deeply identifying some key issues and applying advanced models to shed some light about its characteristics. Like in the case of all cryptocurrencies, important issues analyzed have been what type of asset is bitcoin, for instance comparing it with gold and as a hedging strategy; its speculative nature; identifying factors that affect its pricing and volatility; the presence and meaning of structural breaks; its behavior in relation to the volatility of exchange rates, as well as to other assets and financial markets in general, and its forecasting capabilities. This work stresses recent findings reported in the literature.

Dyhrberg (2016), Bouri *et al*. (2017), Troster *et al.* (2018), Gronwald (2019), and Iheke *et al*. (2020) consider bitcoin as a commodity like gold and adequate as a hedging strategy. Opposite views are those presented by Baur *et al*. (2017), Baur and Hoang (2021) and Klein (2020). These authors find evidence that bitcoin returns follow different volatility process than gold. Additionally, their evidence shows that bitcoin is not correlated to other assets like the dollar.

Considering the high speculative behavior bitcoin, some research assesses bitcoin volatility determining the presence of structural breaks (Baur *et al*., 2019; Sosa *et al*., 2019; Mensi *et al*., 2019; Ardie *et al*., 2019; Rojas and Coronado, 2020; Vo *et al*., 2021). Their studies confirm the presence from two to five structural breaks which impact the volatility of bitcoin.

Regarding bitcoin and exchange rates, Naimy and Hayek (2018) and Szetela *et al*. (2016) forecast the volatility of the Bitcoin/USD exchange rate. Naimy and Hayek (2018) from the in-sample Bitcoin/USD exchange rate returns and both in-sample and out of sample volatility is calculated. Szetela *et al*. (2016) determine the relationship between the exchange rate for bitcoin to the leading currencies such as Dollar, Euro, British Pound and Chinese Yuan and Polish zloty as well.

Forecasting volatility of bitcoin has also been an important area of research (Liang *et al*., 2020; Katsiampa *et al*., 2019a; Urquhart, 2017). Liang *et al*. (2020) investigate which predictor is better for bitcoin volatility from the aspects of in-sample and out-of-sample in a high-speed changing world. In turn, Katsiampa *et al*. (2019a) examine the conditional Dynamic volatility dynamics and conditional correlations bitcoin-ether, bitcoin-litecoin, and ether-litecoin, employing three pair-wise bivariate BEKK models. Urquhart (2017), testing both volatility and forecasting reports no evidence of the leverage effect in Bitcoin. He also examines the volatility of bitcoin and exploring the forecasting ability of GARCH models and Heterogenous Autoregressive (HAR) models in the bitcoin market. He reports that HAR models are superior in modelling bitcoin volatility vis-à-vis conventional GARCH models.

Furthermore,Tiwari (2019) investigate and compare bitcoin and Litecoin volatility using a large number of generalized autoregressive conditional heteroskedastic (GARCH) and stochastic volatility (SV) models. The comparison of GARCH models with GARCH-GJR models reveals that the leverage effect is not significant for cryptocurrencies, suggesting that these crytocurrencies do not behave like stock prices. Köchling *et al.* (2020) analyze the quality of Bitcoin volatility forecasting of GARCH-type models. Applying different volatility proxies and loss functions, they construct model confidence sets and find them to be systematically smaller for asymmetric loss functions and a jump robust proxy; their results signal that 88 out of 148 models are never outperformed.

Finally, an important issue explored regarding bitcoin and cryptocurrencies is their association with other financial markets. Stavroyiannis and Babalos (2017) explore the dynamic properties of bitcoin and the S&P index employing several univariate and multivariate GARCH models, and vector autoregressive specifications. According to the evidence, bitcoin does not actually hold any of the hedge, diversifier, or safe-haven properties; rather, it exhibits intrinsic attributes not related to US market developments. Bouri *et al*. (2018) examine the relations between bitcoin and conventional investments concentrating their analysis on return and volatility spillovers between this cryptocurrency and four asset classes (equities, stocks, commodities, currencies, and bonds) in bear and bull market conditions.

Finally, Lopez-Cabarcos *et al*. (2021) explore bitcoin behavior and the effect that investor sentiment, S&P 500 returns, and VIX returns have on Bitcoin volatility. According to their evidence bitcoin volatility is more unstable during speculative periods. Considering sable periods, Standrd & Poor 500 returns, VIX returns, and sentiment influence the volatility of bitcoin.

Most studies have applied GARCH-family models attempting to capture time varying bitcoin volatility. Although this approach has produced effective and fructiferous results, many works have been excessively concerned with comparing results among a considerable set of GARCH alternatives. Most favored models include: The GARCH, EGARCH, TGARCH, GJRGARCH, APARCH, NGARCH, CGARCH, IGARCH, HARCH, ST-GARCH, MS-GARCH, HYGARCH and SVR-GARCH.

However, more complex models like FIAPARCH, ARFIMA-FIGARCH, GARCH-MIDAS, BEKK-GARCH, VAR-BEKK-GARCH and copula-ADCC-EGARCH have been also employed (Chan and Grant, 2016; Kyriazis, 2021). Results have been positive. The evidence gathered denotes the presence of exponential, threshold, asymmetric, component, power, regime switching, homogeneous autoregressive and even more complex behavior in the volatility of bitcoin and cryptocurrencies under scrutiny in general. Finally, it must be acknowledged that the most sophisticated GARCH methodologies offer more robust explanations about the abrupt peaks and valleys of digital currencies market values (Kyriazis, 2021).

Summing up, volatility analysis of bitcoin has been approached mostly employing different GARCH models. However, despite the effectiveness of GARCH models some authors argue that Stochastic Volatility and GARCH models are inefficient because they are static in nature and ignore the intraday trajectory of an asset. This is an important disadvantage considering only closing prices and neglecting to assess possible abrupt changes that may take during the day (Alizadeh *et al.*, 2002; Brandt and Diebold, 2006; Li and Hong, 2011).

This limitation has induced volatility research to apply high frequency data. Using the high-frequency data of Bitcoin at different time scales Zargar and Kumar (2019), investigates the long memory characteristics of the unconditional and conditional volatilities of Bitcoin local Whittle (LW) estimator, the exact local Whittle (ELW) estimator and the ARMA–FIAPARCH model. The evidence indicates that the long memory parameter is significant and quite stable for both unconditional and conditional volatility measures across different time scales. Further, studying the long memory characteristics of the unconditional and conditional ‘‘realized’’ volatilities of Bitcoin long memory is found to be significant and stable.

Peng *et al*. (2018), Katsiampa *et al.* (2019a) extend this line of research. Peng *et al*. (2018) assess the predictive performance about the volatility of three cryptocurrencies and three currencies with recognized stores of value using daily and hourly frequency data. Their evidence proved that SVR-GARCH models managed to outperform GARCH, EGARCH and GJR-GARCH models with Normal, Student’s *t* and Skewed Student’s *t* distributions.

Similarly**,** Katsiampa *et al*. (2019b) employ the Diagonal BEKK and Asymmetric Diagonal BEKK methodologies to intra-day hourly closing price of cryptocurrency for one hour data for eight cryptocurrencies. The work examines not only conditional volatility dynamics of major cryptocurrencies, but also their volatility co-movements. The results reveal that all conditional variances are unquestionably influenced by both previous squared errors and past conditional volatility. Katsiampa *et al.* (2019b) also prove that the conditional covariances are significantly affected by both cross-products of past error terms and past conditional covariances, implying strong interdependencies between cryptocurrencies. Finally, their work also demonstrates that the Asymmetric Diagonal BEKK model is a superior choice of methodology, with the evidence suggesting significant asymmetric effects of positive and negative shocks in the conditional volatility of the price returns of all of their investigated cryptocurrencies, while the conditional covariances capture asymmetric effects of good and bad news accordingly.

Further, Eross *et al*. (2019) analize the intraday variables of the leading bitcoin exchange with the highest information share including 4 years of data. Applying GMT-timestamped tick data aggregated to the 5-mintuely frequency. The evidence confirms that bitcoin returns have increased over time, but trading volume, volatility and liquidity varied substantially during the period analyzed. Eross *et al*. (2019) further found that volume increases throughout the day and falls from around 4 pm until midnight, which is consistent with the intraday patterns found in currency markets.

Wang and Ngene (2020) explore the intraday dynamics and price patterns of seven major cryptocurrencies. They utilize the Bloomberg 15-minute intraday data and apply the Granger Mackey-Glass (M-G) model to analyze the asymmetric and nonlinear dynamic interactions in the first moment using positive and negative returns. Additionally, the bivariate BEKK-GARCH model is employed to identify cross-market volatility shocks and volatility transmissions in the cryptocurrency market. The evidence shows that bitcoin contains predictive information that can nonlinearly predict the performance of other digital currencies when cryptocurrency prices either are rising or declining. The dominant power of bitcoin is confirmed using the intraday data.

Similarly, Bouri *et al*. (2021) apply functional intraday data analysis techniques to examine cumulative intraday return (CIDR) curves. Their empirical evidence shows that bitcoin CIDR curves are stationary, non-normal, uncorrelated, but exhibit conditional heteroscedasticity; however, the projection scores of CIDR curves could be serially correlated during certain periods. Their study also shows the possibility of predicting the CIDR curves of bitcoins based on the projection scores and then assess the forecasting performance.

Imitiaz *et al*. (2019) deal with the lead-lag relationship between the two leading cryptocurrencies, bitcoin and ethereum. Research apropos price leadership between these assets is limited. They use a battery of statistical tests—VECM, Granger Causality, ARMA, ARDL and Wavelet Coherence—to identify price leadership between these two cryptos. They employ daily and hourly data from August 2017 through to September 2018. The empirical evidence basically suggested bi-directional causality between the two assets.

Petukhina *et al*. (2021) analyze high-frequency data of the cryptocurrency market concerning intraday trading patterns related to algorithmic trading and its impact on the European cryptocurrency market. The authors examine trading quantitatives about returns, traded volumes, volatility periodicity, and summary statistics of return correlations to CRIX (Cryptocurrency Index), as well as respective overall high frequency based market statistics with respect to temporal aspects. The evidence provides important insight into a market, where the grand scale employment of automated trading algorithms and the extremely rapid execution of trades might seem to be a standard based on media reports.

Nonetheless, high frequency data analysis does not necessarily offer better volatility estimates. Lyócsa *et al*. (2021) respond to the assertion that high-frequency volatility models outperform low-frequency volatility models stating that such a conclusion is reached when low-frequency volatility models are estimated from daily closing returns. Hence, they study this question considering daily, low-frequency volatility estimators based on open, high, low, and close daily prices. The data sample consists of 18 stock market indices. Their evidence shows that high-frequency volatility models tend to outperform low-frequency volatility models only for short-term forecasts. If the forecast horizon increases (up to one month), the difference in forecast accuracy becomes statistically indistinguishable for most market indices. Similarly, asset allocation based on high-frequency volatility model forecasts does not outperform asset allocation based on low-frequency volatility model forecasts.

In this respect, proposals for the analysis of volatility alternatives to GARCH models and high-frequency models are range models such as the Garman and Klass (GK), model (1980), and Rogers & Satchell range model (1991).[3]

---

[3]*For reviews of range models and their applications in economics and finance, see Chou et al. (2010), and (2015).*

However, range models for variance analysis applied to cryptocurrencies have been practically overlooked. To estimate mean returns and volatility, these models use not only closing prices, but employ opening, closing, maximum and minimum prices. Earlier advances of these models were presented by Feller (1951), who indicated that the asymptotic distribution of the range of sums of independent random variables can be obtained with the theory of the Brownian movement. Parkinson (1980) proposed an estimator for assets with a simple, driftless diffusion process that considers high and low prices, which shows greater efficiency unlike the estimator that only considers closing prices. For their part, Garman and Klass (G&K) (1980) proposed an estimator of variance which is normally distributed when there is no high frequency data, that is, with significant variations in short periods of time. In this series, prices follow a Brownian movement without a drifting term; That is, each price change is independent of previous price changes and volatility of price changes with a constant term.

Based on those advances, Shu and Zhang (2006), Wu and Hou (2020), Lyósca *et al*. (2021), apply range analysis to investigate stock market volatility. Shu and Zhang (2006) examine the relative performance of several historical volatility estimators that incorporate daily trading range. Range estimators show great precision when an asset price follows a continuous geometric Brownian motion. Nonetheless, sharp differences are observed among various range estimators when the asset return distribution involves an opening jump or a large drift. An empirical test using S&P 500 index returns, shows that the variances estimated with range estimators are quite close to the daily integrated variance.

Wu and Hou (2020) advance a component conditional autoregressive range (CCARR) model for forecasting volatility employing a sample of six stock market indexes. The model assumes that the price range comprises both a long-run (trend) component and a short-run (transitory) component, which has the capacity to capture the long memory property of volatility. The model is intuitive and convenient to implement by using the maximum likelihood estimation method. The evidence highlights the value of incorporating a second component into range (volatility) modelling and forecasting. Particularly, the proposed CCARR model fits the data better than a CARR model.

Considering that Engles' Dynamic Conditional Correlation applies entirely closing prices whereas the literature reports that the high and low prices of a given day can be used to obtain an efficient volatility estimation, Fiszeder *et al*. (2019) adopt a model that incorporates high and low prices into the DCC model. They conduct an empirical evaluation of this model on three datasets: currencies, stocks, and commodity exchange traded funds. Regardless of whether they consider in-sample fit, covariance forecasts or value-at-risk forecasts, their model outperforms not only the standard DCC model, but also an alternative range-based DCC model.

Despite the increasing interest in range modeling to estimate volatility of cryptocurrencies few applications are available in the extant literature. No applications of the R&S models are reported in the literature. Garman and Klass (GK) volatility model extended with an asymmetric bilinear Conditional Autoregressive Range (ABL-CARR) model is employed by Tan *et al*. (2020). Examining the volatility of 102 cryptocurrencies their evidence shows volatility persistence and leverage effects which can improve the predictability of volatility, reduce risk and hence lessen the level of speculation in cryptocurrency market.

Volatility estimators based on daily closing prices are inexact. Range-based volatility estimators provide significantly more precision, albeit remain rowdy volatility estimates, which is sometimes ignored when these estimators are used in further calculations. To overcome this limitation Molnár (2021) analyzes properties of these estimators and find that the best estimator is that calculated using the Garman-Klass (1980) model.

Additionally, Molnár (2021) corrects some mistakes in the literature. Applying the Garman-Klass estimator allows him to obtain an interesting result: returns normalized by their standard deviations are approximately normally distributed. This result, which is in line with results obtained from high frequency data but has never previously been recognized in low frequency (daily) data.

The expected values and mean square errors of the Parkinson, Garman–Klass and Rogers–Satchell estimators for the process with a zero drift and a non-zero drift are derived. Moreover, new volatility estimators, are proposed. The considered estimators are applied to the estimation of the volatility of the Polish stock index WIG20.

4. Modeling Considerations

To assess and discuss the need for correct measures of risk for the price valuation of financial actives, this work proposes to measure bitcoin risk employing Rogers and Satchel model to estimate the standard deviation of its return series. It is important to recall that R&S were highly concerned with adequate estimates of volatility for its applications for option pricing.

4.1 Conventional Linear Standard Deviation

We apply the conventional standard deviation formula as a benchmark to compare it with the Rogers and Satchell model. The conventional standard deviation for the series of n prices is:

$$\sigma = \sqrt[2]{\frac{\sum_{t=1}^{n}(r-\overline{r})^2}{n-1}} \qquad (1)$$

4.2 Rogers and Satchel Range Model

Considering the importance of the Rogers and Satchell (1991) model, this work presents it almost fully with a minimum of style revision. R&S present an improved proposal to the Garman and Klass model (1980) by adding a drift term in the stochastic process to be incorporated into a σ (volatility) estimator using closing, opening, maximum and minimum (COHL) prices.

They point out that the price behavior of a stock is commonly modeled as a Brownian movement with a drift c. Constants c and variance (σ) are not known. To use the B&S pricing formula (to whiche must add the emerging crytpocurrencies derivatives), an estimate of the variance, though not of the drift c, is required. The proposed estimator has the merit of not being subjected to any drift c.

R&S (1991) first acknowledge that the log of the price of a security at time t is commonly expressed as a Brownian motion with a drift: ϱB_t + ct , where B is the standard Brownian movement in $\mathbb{R}$ and $c \in \mathbb{R}$, $\sigma \geq 0$ are unknown variables. In the Black & Scholes option pricing model (to which must add the emerging cryptocurrency derivatives) the exact value of c in the logarithmic price process $X_t = \sigma B_t + ct$ is not important, but the variance σ^2 is an explicit part of formula and, in practice, and must first be estimated to use the formula. In common with other σ estimators, the approximation of the true and low values of the Brownian movement a drift by the high and low values of a random walk introduces an error, which B&S categorize as severe. Therefore, they demonstrate how a correction can overcome this error almost completely.

Thus, R&S propose a model to attain an unbiased estimate of the variance σ^2 for any drift ($\hat{\sigma}^2$), employing opening, close, maximum and minimum prices for a specific interval. In this regard, the model considers a random variable with an exponential distribution (S_T) and one for (I_T) the highest and lowest prices, respectively, of the series X_T, where T represents a specific day. Their aim is to correct an analytic quadratic estimator advanced earlier by Garman and Klass (1980). R&S built an estimator in the special case of c = 0 (so that is just multiple Brownian motion).

In the G&K model Rogers and Satchell found a variance estimator among a class of quadratic estimators, which presents two drawbacks: a) The estimator will be biased if used in the case of nonzero c; and b) In simulations the numerical value is not as close to the true value as it should be. The first drawback is expected: The estimator was built on the assumption that c = 0. The second drawback arises because in simulation, one models Brownian motion by a random walk with Gaussian steps. Now the maximum of the random walk will be in general smaller than the maximum of the Brownian motion; in general, one views only the Brownian motion at discrete set of times. Even taken observations at closer and closer intervals of time the Brownian motion wriggles very wildly (Garman and Klass, 1980; Beckers, 1983).

If one fully looks at the X sample path, one could infer σ^2 from the quadratic variation, but this is not useful for an actual observer who will at best see the price in a very close time sequence. An estimator of σ^2 practically useful will employ only a small, easily available information; and the most readily available information of a trading day of a stock are the opening and closing prices, along with the highest and lowest prices of the day. Usually, the number of shares traded is also readily available.

Let $S_t \equiv sup\{X_u : u \leq t\}$, $I_t = inf\{X_u : u \leq t\}$ and define the estimator

$$\hat{\sigma}^2 \equiv S_1(S_1 - X_1) + I_1(I_1 - X_1) \qquad (2)$$

What Rogers and Satchell want to prove is:

$$E[S_t(S_t - X_t) + I_1(I_1 - X_1)] = \hat{\sigma}^2 t \qquad (3)$$

What stands out is that the right side is independent of c. Therefore, the estimator works correctly for c values other than zero, getting round drawback (a). R&S consider non significative the pay for this in the case of c = 0, where the Garman and Klass estimator is optimal. If $\hat{\sigma}^2_{GK}$ is the G&K estimator,

$$\hat{\sigma}^2_{GK} \equiv k_1(S_1 - I_1)^2 + k_2\big(X_1(S_1 - I_1)\big) - 2I_1S_1) - k_3X_1^2$$

where

$$k_1 = 0.511, \qquad k_2 = 0.019, \qquad k_3 = 0.383$$

then

$$var(\hat{\sigma}^2) = 0.331\sigma^4$$

$$var(\hat{\sigma}^2_{GK}) = 0.27\sigma^4$$

This gets around (a) but the problem for case (b) remains. Therefore, Roger and Satchell propose a correction to the estimators that works effectively when performing the simulations. However, first, they proceed presenting an unbiased estimator of σ^2. For this purpose, let T be an exponential random variable with meanλ^{-1}, independent of B (Brownian movement previously described).Then the probability distribution of S_T is again exponential with parameter $\alpha = (\sqrt{c^2 + 2\lambda\sigma^2} - c)/\sigma^2$, and the probability distribution of $-I_T$ is exponential, with parameter $\beta = (\sqrt{c^2 + 2\lambda\sigma^2} + c)/\sigma^2$. Now it is considered one of the characteristics of the classic Wiener-Hopf factorization of the Lévy X process that S_T and S_T - S_T are independent and $S_T - X_T$ same probability distribution as $-I_T$.

Therefore,

$$ES_T(S_T - X_T) = ES_T E(S_T - X_T) = -ES_T EI_T = \frac{1}{\alpha\beta} = \frac{\sigma^2}{2\lambda}$$

But $ES_T(S_T - X_{tT}) = \int_0^\infty \lambda e^{-\lambda t} dt\, ES_t(S_t - X_t)$ so inversion of the Laplace transform gives

$$ES_t(S_t - X_t) = \sigma^2 t/2$$

and a symmetric argument gives, $EI_t(I_t - X_t) = \frac{\sigma^2 t}{2}$, from which (3) follows.
To compute the variance of this estimator, it is necessary to calculate

$$E(S_T - X_T) + I_T - X_{T)})^2$$

$$E = ES_T^2(S_T - X_T)^2 + 2ES_T I_T(S_T - X_T)(I_T - X_T) + EI_T^2(I_T - X_T)^2$$

$$= 2S_T^2 E(S_T - X_T)^2 + 2ES_T I_T(S_T - X_T)(I_T - X_T)$$

$$= \frac{8}{\alpha^2\beta^2} + 2ES_T I_T(S_T - X_T)(I_T - X_T)$$

since S_T y $S_T - X_T$ are independent exponentials of parameters alfa and beta, respectively;

$$= \frac{2\sigma^4}{\lambda^2} + 2ES_T I_T(S_T - X_T)(I_T - X_t).$$

The cross-moments are hard to compute in the case of a nonzero drift, but in the case of c = 0, Garman y Klass have computed enough of the moments to evaluate these; the answer $var(\hat{\sigma}^2) = 0.331\sigma^4$ comes from applying this. On the other hand, $L^2 triangle\ inequality$ gives a quick estimate for EY^2, where $Y = Y_1 + Y_2 = S_1(S_1 - X_1) + I_1(I_1 - X_1)$, because, as follows easily from the joint law of S_T and $(S_T - X_T)$, $EY_1^2 = EY_2^2 = \frac{\sigma^4}{2}$;
therefore $EY^2 \leq 2\sigma^4$ and $var(\hat{\sigma}^2) \leq \sigma^4$. Furthermore, this bound is, of course, valid for any drift c, and not just c = 0, which is assumed in the exact computation above.

Very important, now R&S proceed to propose a correction which largely overcomes the snag (b) previously identified. The size by which the random walk simulation underestimates S_1 depends on the fitness of the mesh chosen; the more steps taken by the random walk in the time Interval [0,1], a better approximation to S_1 will be obtained. Initially, it is assumed that the number of steps taken by the random walk is known during [0.1], and that it will take $h \equiv 1/N$. Therefore, if X is the log-price process, it can be assumed that the maximum and minimum of $(X_{kh}: 0 \leq k \leq N)$ and X_1, and S_1 must be estimated from this information.

Reviewing this situation, if S_T denotes the maximum of X by time 1, and S denotes the maximum of the embedded random walk, then:

$$S_1 = S + \Delta,$$

hence

$$S_1(S_1 - X_1) = \Delta^2 + \Delta(2S - X_1) + S(S - X_1)$$

where Δ represents the time interval that the random walk takes in the time interval [0.1]. Similarly, if I denote the minimum of the random walk by time 1, then $I_1 = I - \tilde{\Delta}$ where $\tilde{\Delta}$ has the same probability distribution as Δ, and so:

$$I_1(I_1 - X_1) = \tilde{\Delta}^2 - \tilde{\Delta}(2I - -X_1) + I(I - X_1).$$

Thus the estimator $(\hat{\sigma}^2)$ is:

$$(\hat{\sigma}^2 = \left(\Delta^2 + \tilde{\Delta}^2\right) + \Delta(2S - X_1) - \tilde{\Delta}(2I - X_1) + S(S - X_1) + I(I - X_1) \quad (4)$$

Subsequently, Rogers & Satchell show that $E\Delta \doteqdot a\sigma\sqrt{h}$ for some α $(in\ fact,\ \alpha \equiv \sqrt{2\pi}\left[\frac{1}{4} - \frac{\sqrt{2}-1}{6}\right])$, and that $E\Delta^2 \doteqdot b\sigma^2 h$ for some other constant b (in fact, $b \equiv ((1 + 3\pi/4)\ /\ 12)$.

Therefore, they propose to use the estimator $(\hat{\sigma}_h)$, where $(\hat{\sigma}_h)$ is the positive root of the equation

$$\hat{\sigma}_h^2 = 2b\hat{\sigma}_h^2 h + 2(S - I)a\hat{\sigma}_h\sqrt{h} + S(S - X_1) + I(I - X_1) \quad (5)$$

Obtained from (4) by replacing Δ, $\tilde{\Delta}$, Δ^2 and $\tilde{\Delta}^2$ by their expected values. Finally, the values of *a* y *b* must be calculated.

On the other hand, suppose that the maximum value of S taken by the random walk $(X_{kh})_{k=1}^{N}$ is x achieved at time *t*, a multiple of *h*. Then ignoring the possibility that *X* may take a value greater than x outside of the interval $[t - h, t + h]$, so that the underestimate $[\Delta \equiv S_1 - S = S_1 - x]$ is assumed to be well approximated by $sup\{X_s: |t-s| \le h\} - x$. If $H_x \equiv inf\ \{u: X_u = x\}$, the following it is obtained:

$$\frac{P[H_x \in ds|t-h < H_x < t, X_t = x]}{ds} = \frac{h_s(s)p_{t-s}(0)}{\int_{t-h}^{t} h_x(u)p_{t-u}(0)du} \tag{6}$$

where $p_t(\cdot)$ is the transition density for X, and $h_x(\cdot)$ is the density for H_x. Since the interval $[t-h, t]$ is typically small, R&S assume that $h_x(s)$ is effectively constant through this Interval and since

$$p_{t-s}(0) = exp\ (-c^2(t-s)/2\sigma^2)/\sqrt{2\pi\sigma^2(t-s)}$$

is approximately $2\pi\sigma^2(t-s)^{-1/2}$, they assume that the conditional density (16) can be approximated by:

$$P(t - H_x \in du \,|\, t-h < H_x < t, X_t = x)/du \doteqdot (4uh)^{-1/2} \tag{7}$$

Now considering the part of the path of X between $H_x = t - u$ and t. This is a Brownian bridge of duration u, and as such can be represented as:

$$Y_s = \sigma\left(1 - \frac{2}{u}\right) W\left(\frac{us}{u-s}\right), \qquad 0 \le s \le u, \tag{8}$$

where W is a standard Brownian motion (See: Rogers and Williams Theorem; Rogers and Williams (1987)). The distribution of the maximum of Y is easy to compute:

$$P[Y_s > y\ para\ 0 \le s \le u] = P[\sigma W_v > \frac{y(u+v)}{u} for\ some\ v \ge 0]$$

$$= P\left[W_v - \frac{yv}{u\sigma} > \frac{y}{\sigma}\ fgor\ some\ v \ge 0\right]$$

$$(= exp\left(-\frac{2y^2}{u\sigma^2}\right)$$

by the well-known result of the law of the maximum of a downward -drifting Brownian motion.

Thus, if $Z \equiv sup\{X_s: t-h \le s \le t\} - x$, the following can be obtained.

$$P(Z > \alpha | t-h < H_x < t, X_t = x)$$

$$= \int_0^h e^{-(2\alpha^2)/(u\sigma^2)} P[t - H_x \in du | t - h < H_x < t, X_t = x]$$

$$= \int_0^h e^{-(2\alpha^2)/(u\sigma^2)} \frac{du}{(4uh)^{1/2}} \qquad (9)$$

$$= \int_{h^{-1}}^{\infty} e^{-2\alpha^2 s/\sigma^2} \frac{ds}{(4hs^3)^{1/2}}$$

Therefore, R&S deduce:

$$E(Z|t - h < H_x < t, X_t = x) = \frac{\sigma(2\pi h)^{\frac{1}{2}}}{8} \qquad (10)$$

Now determining that $Z' = sup\{X_s : t \leq s \leq t + h\} - x$, then the situation for Z' is essentially the same as for Z, and Z' are independent conditional on $X_t = x$ and $X_s < x$ for $|s - t| > h$. The same approximation to the distribution of Z', can be used, so R&S assume that Z and Z' are independent, with distribution given by (19).

The amount by which the random walk maximum underestimates the maximum of X is thus (approximately) $Z \vee Z'$, whose mean value is computed by calculating:

$$E(Z \wedge Z') = \int_0^{\infty} da \int_{h^{-1}}^{\infty} e^{-\frac{2\alpha^2 s}{\sigma^2}} \frac{ds}{(4hs^3)^{\frac{1}{2}}} \int_{h^{-1}}^{\infty} e^{-\frac{2\alpha^2 u}{\sigma^2}} \frac{du}{(4hu^3)^{\frac{1}{2}}}$$

$$= \frac{\sqrt{2\pi}}{16h} \sigma \int_{h^{-1}}^{\infty} ds \int_{h^{-1}}^{\infty} du \big(s^3 u^3 (s + u)\big)^{-\frac{1}{2}}$$

$$= \frac{\sqrt{2\pi h}}{16} \sigma \int_1^{\infty} dv \int_1^{\infty} dy \big(v^3 y^3 (v + y)\big)^{-1/2} \vee$$

$$= \frac{\sqrt{2\pi h}}{16} \sigma \int_0^1 ds \int_0^1 dt (s + t)^{-1/2}$$

$$= \frac{\sqrt{2\pi h}(\sqrt{2} - 1)\sigma}{6}$$

after some calculation. Hence, they obtain

$$E(Z \vee Z') = E(Z + Z' - Z \wedge Z')$$

$$= \sigma\sqrt{2\pi h}\left[\frac{1}{4} - \frac{\sqrt{2}-1}{6}\right] \qquad (11)$$

With this R&S have demonstrated that $E\Delta \doteqdot E(Z \vee Z') = a\sigma\sqrt{h}$, where a, is the constant stated above. It remains to prove that $E\Delta^2 \doteqdot E(Z \vee Z')^2 = b\sigma^2 h$. Recalling the distribution (17) of Z, first R&S compute

$$EZ^2 = \int_0^{\infty} 2\alpha d\alpha \int_{h^{-1}}^{\infty} e^{-\frac{2\alpha^2 s}{\sigma^2}} \frac{ds}{(4hs^3)^{\frac{1}{2}}}$$

$$= \sigma^2 \int_{h^{-1}}^{\infty} \frac{1}{2s} \frac{ds}{(4hs^3)^{\frac{1}{2}}}$$

$$= \frac{\sigma^2 h}{6} \tag{12}$$

and then

$$E(Z \wedge Z')^2$$

$$= \int_0^{\infty} 2\alpha \, d\, \alpha \int_{h^{-1}}^{\infty} e^{-\frac{2\alpha^2 s}{\sigma^2}} \frac{ds}{(4hs^3)^{\frac{1}{2}}} \int_{h^{-1}}^{\infty} e^{-\frac{2\alpha^2 u}{\sigma^2}} \frac{du}{(4hu^3)^{\frac{1}{2}}}$$

$$= \sigma^2 \int_{h^{-1}}^{\infty} \frac{ds}{(4hs^3)^{\frac{1}{2}}} \int_{h^{-1}}^{\infty} \frac{du}{(4hu^3)^{\frac{1}{2}}} \frac{1}{2(s+u)} \tag{13}$$

$$= \frac{\sigma^2 h}{8} \int_0^1 dt \, \frac{\sqrt{st}}{s+t}$$

$$= \frac{\sigma^2 h}{4} \left(1 - \frac{\pi}{4}\right)$$

after some calculation. Hence immediately from (12) and (13),

$$E(Z \vee Z')^2 = \sigma^2 h \left\{\frac{1}{3} - \frac{1}{4}\left(1 - \frac{\pi}{4}\right)\right\} = \frac{\sigma^2 h}{12}\left\{1 - \frac{3\pi}{4}\right\}$$

that is:

$$E\Delta^2 = \sigma^2 \rho_h := \sigma^2 \frac{h}{12}\left\{1 - \frac{3\pi}{4}\right\}$$

R&S acknowledge that this correction can be applied to the Garman and Klass estimator $\hat{\sigma}^2_{GK,h}$, where $\hat{\sigma}^2_{GK,h}$, solves:

$$\sigma^2 = k_1\left(S + I + 4(S - I)a\sigma\sqrt{h} + 2\sigma^2 h(b + a^2)\right) - k_2 X_1(S + I)$$
$$+ 2k_2\left(IS - (S - I)a\sigma\sqrt{h} - a^2\sigma^2 h\right) - k_3 X_1^2 \tag{14}$$

Here, as previously pointed out, the constants k_1, k_2 y k_3, come from the G&K model, while the final expression obtained by R&S for estimating the variance is

$$\hat{\sigma}_t^2 = S_t(S_t - X_t) + I_t(I_t - X_t) \tag{15}$$

In practice, in order to apply the R&S model, first must be considered the maximum (H_t), and minimum (L_t) prices relation to the opening price (O_t). In this case the closing price is not considered because it corresponds to the price of the previous day. At any rate, each instant of the price in the day represents a variation with respect to the opening prices, therefore it is required to capture the total yield on a given t-day as follows:

$$\bar{r}_{RS_t} = \frac{1}{n}\left[\ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{C_t}{O_t}\right) + \ln\left(\frac{L_t}{O_t}\right)\right] \quad t = 1, 2, 3 \tag{16}$$

The above calculation allows to calculate daily logarithmic yields with intraday prices. This exercise is repeated for each of the observations in the sample.

Taking up the expression (16), it is possible to calculate the sums of each day with intraday prices, where the supreme (S_t) corresponds to $\ln\left(\frac{H_t}{O_t}\right)$, the infimum (I_t) to $\ln\left(\frac{L_t}{O_t}\right)$ and (X_t) to $\ln\left(\frac{C_t}{O_t}\right)$ to replace them in equation (15):

$$Adding\ up_{t1} = \ln\left(\frac{H_1}{O_1}\right)\left[\ln\left(\frac{H_1}{O_1}\right) - \ln\left(\frac{C_1}{O_1}\right)\right] + \ln\left(\frac{L_1}{O_1}\right)\left[\ln\left(\frac{L_1}{O_1}\right) - \ln\left(\frac{C_1}{O_1}\right)\right] \tag{17}$$

That calculation captures the magnitude of the variation at the end of a day; these calculations must be applied to each and every observation from $t = 1$ to $t = n$.

Summing up, the R&S variance estimator, employing intraday (OCHL) prices, can be expressed as follows:

$$\hat{\sigma}_{RS}^2 = \frac{1}{N}\sum_{t=1}^{n} \ln\left(\frac{H_t}{O_t}\right)\left[\ln\left(\frac{H_t}{O_t}\right) - \ln\left(\frac{C_t}{O_t}\right)\right] + \ln\left(\frac{L_t}{O_t}\right)\left[\ln\left(\frac{L_t}{O_t}\right) - \ln\left(\frac{C_t}{O_t}\right)\right] \tag{18}$$

5. Empirical Evidence and Discussion

5.1 Basic Statistics

Variables included bitcoin opening, closing, high and low daily prices for the period November 11, 2013 to March 1, 2020 a total of 2286 observations, using information from *CoinmarketCap*. Data beyond this date because impacts of the Covid-19 on bitcoin volatility behavior needs to be examined employing a comparative approach concerning periods prior, during, and after the crisis. An analysis and discussion of the basic statistics follow.

Table 1. *Descriptive statistics, bitcoin using closing prices*

Mean	Median	Maximum	Minimum	Stand. Dev.	Skewenes	Kurtosis	Jarque Bera
0.001405	0.001100	0.248400	-0.345400	0.0442667	-0.260373	10.511821	5409.696

Source: *Own elaboration.*

Table 1 shows the basic statistics of the series. Essentially, the mean is positive, but the maximum and minimum returns confirm the high swings of bitcoin prices and returns, proper of a speculative market. In fact, regarding volatility, the daily standard deviation, estimated with closing prices, is high, 4.27% away from mean returns. Moreover, this estimator has been calculated exclusively with closing prices using the traditional formula (equation 1) which assumes normality.

This assumption leads to the presence of a Brownian motion characterized by a random walk with improper smaller steps, leading to an estimator underestimation as pointed out by R&S (1991) in relation to snag (b) identified in the creation of their range model. Finally, the bitcoin series under examination reveal a significative skewness to the left, and a very high peaked distribution. Consistent with these findings, the lack of normality is severe as shown by the Jarque-Bera probability. All these findings suggests that a better volatility estimation can be obtained by range estimators because they soften the jumpy random walk captured with only closing prices.

Figures 1 and 2 complement the above analyses. Figure 1 summarizes the behavior of both prices and logarithmic returns employing closing prices of bitcoin for the period under study. It can be observed that the series can characterized with high volatility levels, particularly during 2017, the bitcoin reached prices nearing $20,000 which was then considered as a bitcoin speculative bubble. Although during the following years it did not reach similar increases, for the second semester of 2019 also occurred high volatility levels. Regarding returns, their fall coincided with the volatility trends shown in Figure 1, they are characterized by high peaks, as well as the presence of volatility clusters.

Figure 1. *Bitcoin prices and logarithmic returns - closing prices*

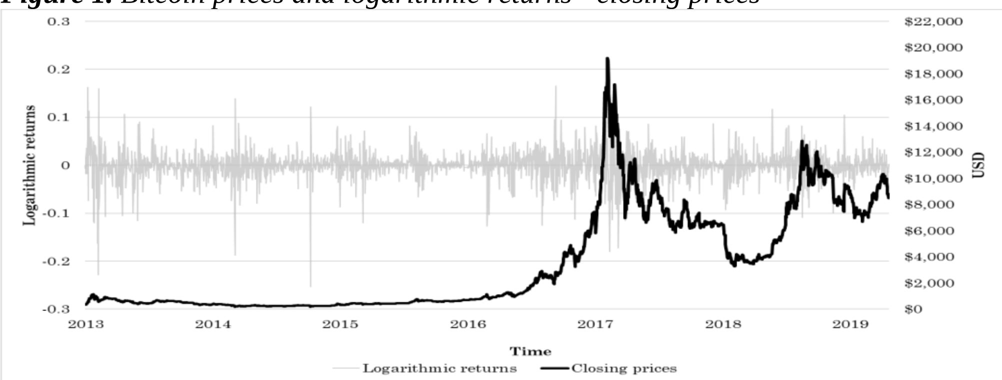
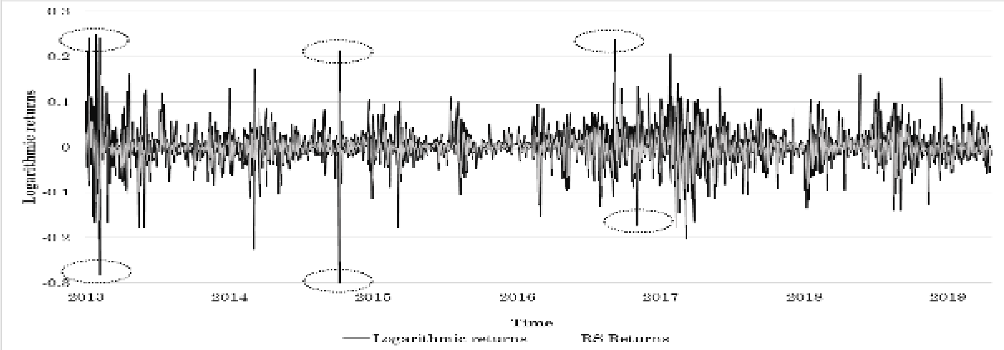


Figure 2. *Bitcoin logarithmic returns Closing and intraday prices*



Source: *Own elaboration.*

Figure 2 shows a decrease in yields when taking different times in the day (gray space). A separation of the charts on the yields was carried out to highlight the results obtained. As shown in Figure 2, there are three observations showing a significant reduction which, considering only logarithmic yields with closing prices, were placed in the range (0.2, 0.3) and with intraday prices the variation decreases to the band with the limits (0.1, 0.2); also, in the case of negative yields, there is also a reduction.

Finally, to prior to examining the R$R model is necessary to test whether the bitcoin series analyzed are stationary or not. Table 2 summarize the results obtained, for both closing and intraday series applying the Dickey Fuller Augmented Test (ADF). The evidence suggests that the null hypothesis is rejected since the absolute values of the ADF statistic in both series are higher than MacKinnon's critical values at all significance levels. Hence, the first difference series are stationary. All root test analyses, including those o price levels, are available upon request, contacting the authors.

Table 2. *Augmented Dickey-Fuller Test*

	Cierre	Intradía
Estadístico ADF	-48.38891	-50.20506
Nivel de significancia	Valores críticos	
1%	-3.433016	-3.433016
5%	-2.862604	-2.862604
10%	-2.567382	-2.567382
Probabilidad	0.0001	0.0001

Source: *Own elaboration using data from CoinmarketCap, 2021.*

5.2 Rogers and Range Application and Discussion

For the estimation of the logarithmic performance of a given day, applying R&S model, the natural logarithm of the opening price and closing price was calculated.

For the first observation is three logarithmic returns prices for each day in the sample. Additionally, the average return can be calculated from the logarithmic yields of each of the observations, as follows

It is possible to contrast the logarithmic yields of closing prices with the behavior from the maximum, minimum and closing prices. To do this, the average $\bar{r}_{RS_t}$ is calculated, with the logarithmic yields of each of the observations as follows:

$$\bar{r}_{RS} = \frac{1}{n}\sum_{n=1}^{t}\left[\ln\left(\frac{H_n}{O_n}\right) + \ln\left(\frac{C_n}{O_n}\right) + \ln\left(\frac{L_n}{O_n}\right)\right]$$

Where $\ln\left(\frac{H_n}{O_n}\right)$, is the logarithmic return corresponding to the highest price relative to the opening price; $\ln\left(\frac{C_n}{O_n}\right)$, is the logarithmic performance of the closing price relative to the opening price; and $\ln\left(\frac{L_n}{O_n}\right)$ is the logarithmic performance of the minimum price relative to the opening price. The above result allows to compare with the calculated yield with the closing price in t-1. Then, the R&S $\hat{\sigma}_{RS}^2$ formula must be applied:

$$\hat{\sigma}_{RS}^2 = \frac{1}{N}\sum_{n=t-N}^{t}\ln\left(\frac{H_n}{O_n}\right)\left[\ln\left(\frac{H_n}{O_n}\right) - \ln\left(\frac{C_n}{O_n}\right)\right] + \ln\left(\frac{L_n}{O_n}\right)\left[\ln\left(\frac{L_n}{O_n}\right) - \ln\left(\frac{C_n}{O_n}\right)\right]$$

For comparative purposes Table 3 reports the variance estimated both applying only daily closing prices, employing the traditional variance formula (equation (1)), as well as the estimation employing R&S model. The evidence shows that estimation of variance with the R&S model overcomes the underestimation registered assuming normality applying the conventional variance formula. As previously mentioned, R&S (1991) categorically emphasize that assuming normality leads to the presence of a Brownian motion characterized by a random walk with improper smaller steps, leading to a volatility underestimation. Hence, correcting this shortcoming the R&S estimation is higher. Congruent with R&S original their model provides a reliable risk parameter for the application in the valuation of bitcoin financial and real options which can be extended for the case of other digital currencies and in general for the valuation of derivatives.

Table 3. *Estimated Daily Variances – closing price and R&S returns*

	Closing	**Rogers and Satchell**
Variance	0.00182	0.00196
Stanndard Deviation	0.00182	0.00196

Source: *Own elaboration.*

Estimates included in Table 3 report volatility in terms of daily performance. This is a very useful information for dynamic decision making by large institutional market participants. However, medium, small, and individual investors, for their decisions and reports need information other that daily (weekly, monthly, quarterly, annual).

Table 4 reports the variance of bitcoin in annual terms and for the last quarter of the total period under analysis, that is, December 2019 to March 1, 2020. Annualized information is estimated using the daily variance assuming 360 daily transactions.

Table 4. *Annualized and Quarterly Variance Estimates, Closing Prices and R&S*

	ANNUALIZED		LAST QUARTER	
	Closing	R&S	Closing	R&S
Variance	3.45%	3.71%	7.53%	6.11%
Standard Deviation	1.86%	1.93%	2.74%	2.47%

Source: *Own elaboration.*

These results add more realism to the volatility of bitcoin. Applying both models, in annualized terms, for the period under analysis bitcoin's variance is over three standard deviations from the mean returns, and around 7% for the last quarter. Regarding the standard deviation annualized returns near two percent. In the long run bitcoin volatility been relatively stable compared with the S&P's performance. A ten-year annualized variance for the Standard and Poor Depository Receipts (SPDR) 500 EFT is 1.542%. (yahoo!finance, 2021). This fund traded in the New York Stock Exchange tracks the S&P stock market index.

However, bitcoin high volatility seems associated in the short run with speculative periods marked by either favorable or unfavorable events related to the evolution of this digital currency, which is confirmed by the review of key events surrounding the evolution of bitcoin. In fact, regarding the last quarter of the sample conventional measurements of the variance and covariance of bitcoin are greater than those estimated using the R&S range model. This probably can be attributed to speculation, as well as to outliers present at the beginning of the Covid-19 crisis. This evidence suggests the need to use more advanced GARCH models to capture the complex behavior of bitcoin, particularly during short (speculative) periods.

6. Concluding Remarks

The volatility of bitcoin has been subject to various events related to either its acceptation or rejection as a full financial asset. Considering the complex behavior of that asset's volatility, this work presents and applies Rogers and Satchell range model. A revision of the literature reveals that range cryptocurrencies volatilities have mostly studied using diverse GARCH type but stressing comparative approaches and limited to closing prices series, which might lead to some erroneous volatility estimation by ignoring abrupt intraday changes. To overcome such limitation high frequency data models have been used.

However, high frequency data analysis does not necessarily offer better volatility estimates. Indeed, Lyócsa *et al*. (2021) assert that high-frequency volatility models outperform low-frequency volatility models when low-frequency volatility models are estimated from daily closing returns. However, high-frequency volatility models tend to outperform low-frequency volatility models only for short-term forecasts.

Consequently, to GARCH and high-frequency models range models emerged as practical alternatives. However, range models for variance analysis applied to cryptocurrencies have been practically overlooked. These models have been mainly applied to estimate the volatility of stock markets, exchange rates, and commodity exchange traded funds. Few related studies using range models to estimate cryptocurrencies volatility with these models are reported in the literature. Only two related studies could be identified No applications of the R&S models are reported in the literature, making this work the first to fully acknowledge and employ it. The evidence confirms its usefulness. R&S is an unbiased estimator regardless of drift and allows the error not to be affected by the high and low values of the series.

Thus, the empirical evidence reveals B&S estimations of both the variance and the standard deviation of bitcoin volatility estimations are higher than those estimated using the conventional volatility formula, overcoming underestimation when normality is assumed. Additionally, as expected, bitcoin in the long rung has been higher compared with the S&P's performance. A ten-year variance annualized for the Standard and Poor Depository Receipts Fund is 1.53% versus 1.93% estimated with the R&S model. Finally, the last quarter of the sample conventional measurements of the variance and covariance of bitcoin are greater than those estimated using the R&S range model. This probably can be attributed to speculation, as well as to outliers present at the beginning of the Covid-19 crisis. This evidence suggests the need to use more advanced GARCH models to capture the complex behavior of bitcoin, particularly during short (speculative) periods.

Finally, an important evidence drawn from key events related to the evolution of bitcoin reveals the need for further and improved regulation of cryptocurrencies. which are definitively evolving to stay and most likely will organize in some collaboration nets. Also, countries are contemplating the creation of Central Banks Digital Currencies which will coexist with private cryptocurrencies. An appealing suggestion is the creation of an IMF global currency.

However, negotiations would be very complex and lengthy. A practical solution could be to increment SDRs along with a redistribution of IMF quotas, promoting a greater participation of developing nations. A Strengthened international governance would help to overcome the problems created by the covid pandemic increasing, investments, and growth. All member countries would retain their control of local monetary policies.

Future research should extend R&S model considering leptokurtosis, other moments of the distribution, additional distributions, and short and long run volatility estimates.

References:

Alizadeh, S., Brandt, M.W., Diebold, F.X. 2002. Range-based estimation of stochastic volatility models. Journal of Finance, 57(3), 1047-1091.

Ardia, D., Bluteau, K., Rüede, M. 2019. Regime changes in Bitcoin GARCH volatility dynamics. Finance Research Letters, 266-271.

Baur, D.G., Dimpfl, T., Kuck, K. 2017. Bitcoin, gold and the US dollar - A replication and extension. Finance Research Letters, 25(C), 103-110.

Baur, D.G., Gil-Alana, L.A., Gupta, R., Roubaud, D. 2019. Modelling long memory volatility in the Bitcoin market: Evidence of persistence and structural breaks. International Journal of Finance and Economics, 24(1), 412-426.

Baur, D.G., Hoang, L.T. 2021. A crypto safe haven against Bitcoin. Finance Research Letters, 38, 101431.

Bariviera, A.F., Merediz-Solà, I. 2020. Where Do We Stand in Cryptocurrencies Economic Research? A Survey Based on Hybrid Analysis. Journal of Economic Surveys. https://doi.org/10.1111/joes.12412. Available at: SSRN: https://ssrn.com/abstract=3553071; http://dx.doi.org/10.2139/ssrn.3553071.

Beckers, S. 1983. Variance of security price returns based on high, low, opening and closing prices. Journal of business, 56, 97-112.

Black, F., Scholes, M. 1973. The Pricing of Options and Corporate Liabilities. The Journal of Political Economy, 81(3), 637-654.

Bollerslev, T. 1986. Generalized autoregressive conditional heterokedasticity. Journal of Econometrics, 31, 307-327.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., Hagfors, I.L. 2017. On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? Finance Research Letters, 20, 192-198.

Bouri, E., Das, M., Gupta, R., Roubaud, D. 2018. Spillovers between Bitcoin and other assets during bear and bull markets. Applied Economics, 50(55), 5935-5949.

Bouri, E., Chi-Keunh, M.L., Saeed, T., Wang, S., Zhao, Y. 2021. On the intraday return curves of Bitcoin: predictability and trading opportunities. International Review of Financial Analysis. https://doi.org/10.1016/j.irfa.2021.101784.

Brandt, M.W., Diebold, F.X. 2006. A No-Arbitrage Approach to Range Based Estimation of Return Covariances and Correlations. Journal of Business, 79, 61-73.

Chan, J.C.C., Grant, A.L. 2016. Modeling energy price dynamics: GARCH versus stochastic volatility. Energy Economics, 54, 182-189.

Chou, R.Y., Chou, H., Liu N. (2010) Range Volatility Models and Their Applications in Finance. In: Lee CF., Lee A.C., Lee J. (eds). Handbook of Quantitative Finance and Risk Management. Springer, Boston, MA. https://doi.org/10.1007/978-0-387-77117-5_83.

Chou, Y., Hengchih, H., Liu, N. 2015. Range Volatility: A Review of Models and Empirical Studies, 74, 2029-2050. In: C.F. Lee, J. Lee (eds.), Handbook of Financial Econometrics and Statistics, DOI 10.1007/978-1-4614-7750-1_74. Springer Science & Business Media, New York.

Corbet, S., Lucey, B., Urquhart, A., Yarovaya, L. 2018. Cryptocurrencies as a financial asset:

a systematic analysis. International Review of Financial. Analysis. https://doi.org/10.1016/j. irfa.2018.09.003.

Dyhrberg, A.H. 2016. Bitcoin, gold and the dollar: A GARCH volatility analysis. Finance Research Letters, 16, 85-92.

Eross, A., McGroarty, F., Urquhart, A., Wolfe, S. 2019. The intraday dynamics of bitcoin. Research in International Business and Finance, 49, 71-81.

Feller, W. 1951. The Asymptotic Distribution of the Range of Sums of Independent Random Variables. Annals of Mathematical Statistics, 22(3), 427-432. https://doi.org/10.1214/aoms/1177729589.

Fiszeder, P., Fałdzińsk, M., Molnár, P. 2019. Range-based DCC models for covariance and value-at-risk forecasting. Journal of Empirical Finance, 54, 58-76.

Garman, M., Klass, M.J. 1980. On the estimation of security price volatilities from historical data. Journal of Busines, 53, 67-78.

Gronwald, M. 2019. Is Bitcoin a Commodity? On price jumps, demand shocks, and certainty of supply. Journal of International Money and Finance, 97, 86-92.

Kyriazis, N.A. 2021. A Survey on Volatility Fluctuations in the Decentralized Cryptocurrency Financial Assets. Journal of Risk and Financial Management, 14, 293-339. https://doi.org/10.3390/jrfm 14070293.

Iheke, D.O., Lin, B. 2020. Crude oil price and cryptocurrencies: Evidence of volatility connectedness and hedging strategy. Energy Economics, 87, 104703. https://doi.org/10.1016/j.eneco.2020.104703.

Imitiaz, M., Azhar, M., Syazwan, M., Bin, M., Imitiaz, S. 2019. Lead-Lag relationship between Bitcoin and Ethereum: Evidence from hourly and daily data. Research in International Business and Finance, 50, 306-321.

Katsiampa, P., Corbet, S., Lucey, B. 2019a. Volatility spillover effects in leading cryptocurrencies: A BEKK-MGARCH analysis. Finance Research Letters, 29, 68-74.

Katsiampa, P., Corbet, S., Lucey, B. 2019b. High frequency volatility co-movements in cryptocurrency markets. Journal of International Financial Markets, Institutions and Money, 62, 35-52.

Klein, T., Thu, H.P., Walther, T. 2018. Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance. International Review of Financial Analysis, 59, 105-116.

Köchling, G., Schmidtke, P., Posch, P. 2020. Volatility forecasting accuracy for Bitcoin. Economics Letters, 191, 108836.

Li, H., Hong, Y. 2011. Financial volatility forecasting with range-based autoregressive volatility model. Finance Research Letters, 8(2), 69-76. https://doi.org/10.1016/j.frl.2010.12.002.

Liang, C., Zhang, Y., Feng, M. 2020. Which predictor is more predictive for Bitcoin volatility? And why? International Journal of Economics and Finance. https://doi.org/10.1002/ijfe.2252.

Lopez-Cabarcos, M.A., Pérez-Pico, A.M., Piñeiro-Chousa, J., Šević[c], A. 2021. Bitcoin volatility, stock market and investor sentiment. Are they connected? Finance Research Letters, 38, 101399.

Lyócsa, S., Molnár, P., Výrost, T. 2021. Stock market volatility forecasting: Do we need high-frequency data? International Journal of Forecasting. https://doi.org/10.1016/j.ijforecast.2020.12.001.

Mensi, W., Yahyaee, A.K.H., Kang. S.H. 2019. Structural breaks and double long memory of cryptocurrency prices: A comparative analysis from Bitcoin and Ethereum. Finance Research Letters, 29, 222-230.

Molnár, P. 2021. Properties of range-based volatility estimators. International Review of Financial Analysis, 23, 20-29**.**

Naimy, V., Hayek, M.R. 2018. Modelling and predicting the Bitcoin volatility using GARCH models. International Journal of Mathematical Modelling and Numerical Optimisation, 8(3), 197-215.

Ocampo, J.A. 2021. Point of View – The SDR's time has come: Rethinking the Special Drawing Right could bolster the IMFs role in the global finance safety net. Finance & Development, 56(4), 62-63.

Parkinson. M. 1980. The extreme value method of estimating the variance of the rate of return. Journal of Busines, 53(1), 61-65.

Peng, Y., Melo Albuquerque, P.H., Martins-Camboim de Sá, J.M, Akaishi-Padula, A.J., Montenegro, M.R. 2018. The best of two worlds: Forecasting high frequency volatility for cryptocurrencies and traditional currencies with Support Vector Regression. Expert Systems with Applications, 97, 177-192**.**

Petukhina, A.A., Reule, R.C.G., Härdle, W.K. 2021. Rise of the machines? Intraday high-frequency trading patterns of cryptocurrencies. The European Journal of Finance, 27(1-2), 8-30. DOI: 10.1080/1351847X.2020.1789684.

Rogers, L.C.G., Satchell, S.E. 1991. Estimating Variance From High, Low and Closing Prices. The Annals of Applied Probability, 1(4), 504-512. https://doi.org/10.1214/aoap/1177005835.

Rogers, L.C.G., Williams, D. 1987. Diffusions, Markov Processes, and Martingales. Wiley, Chischester.

Rojas, O., Coronado, S. 2020. A Bayesian study of changes in volatility of Bitcoin. Contaduría y administración, 65(3), 1-13.

Sosa, M., Ortiz, E., Cabello, A. 2020. Financiamiento empresarial con cripto-tokens: Riesgos y su valuación. Working Paper. Programa de Posgrado. Facultad de Ciencias Políticas y Sociales, Universidad Nacional Autónoma de México.

Sosa, M., Ortiz, E., Cabello, A. 2019. Bitcoin Conditional Volatility: GARCH Extensions and Markov Switching Approach. In: J.J. Choi, B. Ozcan (eds.), Disruptive Innovation in Business and Finance in the Digital World. International Finance Review, 20, 201-219. Emerald Publishing Limited.

Shu, J., Zhang, J.E. 2006. Testing range estimators of historical volatility. The Journal of Futures Markets, 26(3), 297-313. https://doi.org/10.1002/fut.20197.

Stavroyiannis, S., Babalos, V. 2017. Dynamic Properties of the Bitcoin and the US Market: Available at: https://ssrn.com/abstract=2966998 or http://dx.doi.org/10.2139/ssrn.2966998.

Szetela, B., Mentel, G., and Gedek, S. (2016). Dependency Analysis between Bitcoin and Selected Global Currencies. Dynamic Econometric Models. Dynamic Econometric Models, vol. 16, pp.133-144.

Tan, S.K., Chan, S.K., HaurNg, K. 2020. On the speculative nature of cryptocurrencies: A study on Garman and Klass volatility measure. Finance Research Letters, 32, 1-5. https://doi.org/10.1016/j.frl.2018.12.023

Tiwari, A.V., Kumar, S., Pathak, R. 2019. Modelling the dynamics of Bitcoin and Litecoin: GARCH versus stochastic volatility models. Applied Economics, 37(37), 4073-4082.

Troster, V., Tiwari, A., Shabbaz, M., Macedo, M. 2018. Bitcoin returns and risk: A general

GARCH and GAS analysis. Finance Research Letters, 30. DOI:10.1016/j.frl.2018.09.014.

Urquhart, A. 2017. The Volatility of Bitcoin. Researh Paper. Available at SSRN: https://ssrn.com/abstract=2921082 /http://dx.doi.org/10.2139/ssrn.2921082.

Vo, A., Chapman, T.A., Lee, Y.S. 2021. Examining Bitcoin and Economic Determinants: An Evolutionary Perspective. Journal of Computer Information Systems, 61.

Wang, J., Ngene, G.N. 2020. Does Bitcoin still own the dominant power? An intraday analysis. International Review of Financial Analysis, 71, 101551

Wu, X., Hou, X. 2020. Forecasting volatility with component conditional autoregressive range model. The North American Journal of Economics and Finance, 51, 101078. https://doi.org/10.1016/j.najef.2019.101078.

Zargar, F.N., Kumar, D. 2019. Long range dependence in the Bitcoin market: A study based on high-frequency data. Physica A: Statistical Mechanics and its Applications, 515, 625-640. https://doi.org/10.1016/j.physa.2018.09.188.

Webpages:

Bank of England. 2021. Bank of England News Release June 7, 21. https://www.bankofengland.co.uk/news/2021/june/new-forms-of-digital-money-discussion-paper-and-summary-of-responses-to-the-discussion-paper-on-cbdc.

BitGreen. 2021. The BitGreen Ecosystem. BitGreen Whitepaper: htps://bitg.org/whitepaper/#Top

CoinMarketCap. 2021. Cryptocurrency Prices, Charts And Market Capitalizations CoinMarketCap Saturday, 30 January 2021 19:55:18, and July 3, 12.47:40 UTC.

ComplyAdvantage. 2021. Cryptocurrency Regulations in Japan: Cryptocurrency Regulations in Japan | ComplyAdvantage Cryptocurrency Regulations in Japan | ComplyAdvantage

Crypto Daily. 2021. Will CBDCs Threaten Cryptocurrencies?: https://cryptodaily.co.uk/2021/03/Will-CBDCs -threaten-cryptocurrencies

Faast. 2021. What the Rise of CBDCs Means for Cryptocurrency: https://medium.com/faast/what-the-rise-of-cbdcs-means-for-cryptocurrency

Google Finance. 2021. NASDAQ: COIN: https://www.google.com/finance/quote/COIN:NASDAQ

Market Insider. 2021. Global central banks are pressing ahead with testing digital currencies, with China leading the way: https://markets.businessinsider.com/currencies/news/global-central-banks-digital-currencies-china-leads-cryptocurrencies- -2021-3-1030249746

Microestrategy. 2021. Microestrategy Completes $1.95 b illion offering of convertible notes at 0% coupon and 50% conversion premium with bitcoin use of funds. Press release.

Microestrategy, February 19, 2021. https://www.microstrategy.com/en/investor-relations/press/microstrategy-completes -1-05b-offering-of-convertible-senior-notes_02-19-2021.

NASDAQ. 2021. Options on Coinbase Global start trading in robust volume: https://www.nasdaq.com/articles/options-on-coinbase-global-start-trading -in-robust-volume-2021-04-20.

Office of the Comptroller of the Currency. 2020. Interpretive Letter #1170. Authority of a National Bank to Provide Cryptocurrency Custody Services for Customers. July 22, 2020: Interpretive Letter 1170, Authority of a National Bank to Provide Cryptocurrency Custody Services for Customers (occ.gov)

Phillips, D. 2020. Five best bitcoin options trading exchanges (updated 2020). Coindiligent. Advertiser Disclosure, April 17, 2020: https://coindiligent.com/bitcoin-options-exchanges

Plublius. 2018. BitCoin Crypto Review. November 3, 2018: https://amorfati0.wordpress.com/2018/11/03/peercoin-crypto-review/

Reuters. 2021a. Analysis: China digital currency trials show threat to Alipay, WeChat duopoly: https://www.reuters.com/technology/analysis-china-digital-currency -trials-show-threat-alipay-wechat-duopoly-2021-04-26/

Reuters. 2021b. El Salvador bonds tumble as investros eye bitcoin use, IMF talks. Reuters. Business News, September 16.2021: https://www.reuters.com/article/el-salvador-debt/el-salvador-bonds-tumble-as-investors- -eye-bitcoin-use-imf-talks-idUSL1N2QI290

Taraldsen, L.E. 2021. Norwegian Oil Billionaire Joins the Bets on Bitcoin The Quint, Bloomberg, March 8, 2021: https://www.bloombergquint.com/business/billionaire-rokke-bets-bitcoin-will-be-on-right-side-of-history

Thellemann, P. 2020. Deribit Review: Fees, Coins, Leverage and More. Coindiligent: https://coindiligent.com/deribit-review

Yahoo! Finance. 2021. Here's how Coinbase stock performed on its first full day of trading: https://finance.yahoo.com/news/how-coinbase-direct-listing-stock-cryptocurrency-bitcoin-is-doing-debut- 134743395.html-is-doing-debut-1

Yahoo!finance. 2021. SPY 435.52 4.60 1.07% : SPDR S&P 500 ETF Trust - Yahoo Finance Wall Street Journal. 2020. Why Central Banks Want to Create Their Own Digital Currencies Like Bitcoin, October 21, 2020. https://www.wsj.com/articles/why-central-banks-want-to-create-their-own-digital

Wall Street Journal. 2021. El Salvador Becomes First Country to Approve Bitcoin as Legal Tender, June 9, 2021. https://www.wsj.com/articles/el-salvador-becomes-first-country-to-approve-bitcoin-as-legal-tender-11623234476