International Journal of Finance, Insurance and Risk Management
Volume XIII, Issue 2, 2023

pp. 167-188

Rational Investors or Rational Expectations in Efficient Market Hypothesis?

Submitted 12/04/23, 1st revision 03/05/23, 2nd revision 12/05/23, accepted 30/06/23

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Abstract:

Purpose: *The main goal of this research is to determine whether the tests of the Effecient Market Hypothesis, accepting the neoclassical rationality of investors, can be carried out on data (prices, rates of return) only from the immediate market*

Design/Methodology/ Approach: *The article defines a valuation model by rational investors in which market information is constantly updated and used to determine current and future stock prices. Valuations are made using the Discounted Cash Flow (DCF) method, which is the neoclassical core of financial theory. Expected future prices are a continuous function of time (current research focuses on current value with discrete time analysis). The model was used to test the validity of the efficient market hypothesis (EMH) based on historical and current market price data.*

Findings: *Incorporation of neoclassical investor rationality leads to the conclusion that EMH tests based only on past and present data may give erroneous results. Bypassing investors' expectations about future makes it difficult to see a possible new price trend created by the arrival of new informatoion to capital market. This may result in incorrect assumptions about the randomness of immadiate market price deviations (return rates) from equilibrium and not to mention that prices contain new information. Then, over time, consecutive instantaneous prices spread around a new but unaddressed trend may be mistakenly assessed as non-random deviating from the trend so far, which may result in the market becoming ineffective in information, despite its information efficiency. Excluding from EMH testing expected future value of the variables is contrary to the principle of investor rationality. Moreover, the deletion of forward data makes it impossible to see the long-term price trend. This distorts the established short-term trend, in particular for spot prices ending the trial. Due to the disturbance in the estimation of the trend, the parameters of random deviations, including autocorrelation, change. This may lead to a faulty conclusion about the market efficiency and the correctness of the asset pricing model.*

Practical Impications: *The tests of the efficient market hypothesis, according to the principle of investor rationality, should take into account the future data held by investors, for example included in forward market prices, or published forecasts of fundamental values.*

Originality/Value: *The obtained results inspire further research into the trends of stock prices and the characteristics of the random noise of prices which are of a rational nature. Deviations from the rational model may constitute a measure of the investor's irrationality and market information inefficiency. This will be the subject of subsequent publications.*

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Keywords: *Effecient market hypothesis, weak, medium and strong EMH version, rational investors, rational expectations, valuation model, asset pricing model.*

JEL codes: *G11, G12, G13.*

Paper Type: *Theoretical article.*

1. Introduction

The effecient market hypothesis (EMH) has been the subject of disputes among investors and theoreticians for many years. It puts into question the rationality of investor behavior and the legitimacy of using technical, fundamental and, in extreme cases, information inaccessible to others (Radcliffe, 1982, p. 621). On the other hand, numerous studies show that there is information that allows to achieve above-average rates of return, providing evidence for inaccuracy of the market efficiency theory (Haugen, 1999).

Thus, the valuation of shares and other financial instruments contains errors (anomalies), and an investor who can detect them can achieve above-average returns. As pointed out by R. Ślepaczuk, the efficiency of capital markets in the informative sense is related to allocation efficiency and transaction efficiency (Ślepaczuk, 2006, pp. 1-2). In these aspects, the market's efficiency is confirmed by the string of rational investors' reactions to information coming from the market. The capital market is efficient in allocation sense if it creates the possibility of raising capital only for those market participants who are able to locate it in a manner that brings the highest rate of return.

An efficient market in the transactional sense, in turn, means a situation in which competition between intermediaries operating on the market enforces a reduction in transaction costs and, as a consequence, leads to almost immediate conclusion of purchase and sale transactions of instruments listed on the market (Gurgul, 2006, p. 14).

According to the efficiency theory, companies listed on efficient stock markets are valued taking into account all publicly available information of both a technical and a fundamental nature. The price change is possible only in the case of new information appears on the market, investors' consent as to its significance in the shares valuation and immediate use of it in a linear relation. Market participants, acting as a collective, continuously and proportionally set a new equilibrium rate (Gabryś, 2008, p. 218). Theoretically, investors who buy or sell securities in an efficient market should not be able to achieve above-average returns, because securities are always priced at the right level.

EMH testing involves examining random deviations around the price trend (rate of return) determined on the basis of the adopted asset pricing model. The asset pricing

model is rational, which means in particular that the investor takes into account all available information. Meanwhile, a significant part of the research testing the EMH hypothesis neglects data from the forward market, based on the tradition going back to the work of Samuelson and Fama.

We hypothesize that omission of forward prices from EMH testing leads to an increase in the risk of EMH test result errors. The main goal of the research is to determine whether the tests of the Effecient Market Hypothesis, accepting the neoclassical rationality of investors, can be carried out on data (prices, rates of return) only from the immediate market.

The auxiliary research goals are as follows:

1. Defining a rational continuous valuation model based on discounted expected cash flows, which will result in expected future prices as a function of time;
2. Veryfying the correctness of the results generated by the defined model in the classic case of the exponentially changing dividend (Gordon-Shapiro model) in order to obtain a dynamic valuation (valuation as a function of time).

We formulate the following research hypotheses:

1. EMH testing based on data from the immediate market ensures the unbiased and completeness of the model used for EMH testing,
2. The appearance of non-random price deviations from the forward market from theoretical prices determined by the correct model until the disclosure of information means that the disclosure of new information in prices requiring the model's reliance (alternative hypothesis: the market is inefficient in information).

This article adopts the following structure: the first section presents the Fama's theory of efficient markets with a discussion about the different varieties of efficient market hypothesis and newer EMH approaches. The next section presents the model of asset valuation by rational investors based on the discounted cash flow method.

The valuation model covers the entire pricing process:

1. The appearance of information that carries the data used in the valuation process.
2. Decoding information by investors.
3. The formation of investors' expectations about prices.
4. Setting the equilibrium price on investors' expectations.

The model was defined in continuous time. This will allow the use of functions and a different-integer calculation in the analysis, which will facilitate the planned calculations in relation to the time-consuming calculations necessary when using discrete time. The model is used to examine the valuation based on the assumptions

of the Gordon-Shapiro model, in order to obtain a forward price as a function of time (dynamics). Then, we discussed the problem of not considering the values of fundamental variables and forward prices while testing the EMH hypothesis, which may lead to an incorrect assessment that the market is informative inefficient.

2. The Effective Market Hypothesis - The Theory of Fama and Further Approaches

The theory of market efficiency was developed by Eugene Fama, who in his first works defined efficient market as a market that quickly and unambiguously adapts to new information (Fama, 1970, 384-417). The modern and more mature understanding of the efficient market according to Fama assumes that it is a „market with a large number of rational, guided by the principle of profit maximization and actively competing market participants, each trying to predict the future market value of individual securities and where important current information is almost costlessly available to all participants. On the efficient market, competition between intelligent participants leads to a situation where the current prices of individual securities at any given time reflect information related to events that have taken place and events that are expected to appear on the market in the future" (Fama, 1991, pp. 1575-1617).

In the newer literature increasily cited is the definition of an effective market proposed by B. Malkiel which says that "the capital market is effective when it reflects fully and correctly the appropriate information on the price movements. Formally, the market is called effective in relation to the set of informaction data, if the prices of assets do not change when information from this set is provided to all market participants. Effectiveness also implies that it is not possible to make profits from investments only on the basis of information from the Dt collection" (Malkiel, 1992 cited in Gurgul, 2006, p. 18). Therefore, markets are effective when any publicly available information is discounted in a price, and share prices usually correspond to the value of the company, and they are not systematically overestimated or underestimated.

The efficient market hypothesis is built on three assumptions (Szyszka, 2003, p. 13):

1. Investors make a reasonable valuation of financial assets and seek to maximize profits, where the investor's rational behavior is defined as the ability to value listed instruments based on all available information and their appropriate valuation (Gabryś, 2008, p. 481; Riepe, 1998). In addition, the expectations of investors are assumed to be homogeneous.
2. If investors turn out to be irrational, their individual actions are random and neutralize each other, without affecting the share prices.
3. If, within certain limits, investors behave irrationally, but in a similar way, which could affect the level of prices, then they come across the market for

rational investors who, by means of arbitration, eliminate the influence of investors acting irrationally.

The author of the efficient market hypothesis also formulated conditions that in his opinion are sufficient to define the market as efficient. First, universal access to information for all market participants is ensured. Second, there are no transaction costs in stock trading. Third, capital market participants agree on the impact of new information on securities prices. H. Gurgul points out that no real capital market fulfills all these conditions and that these are not necessary but only sufficient conditions. In his opinion the degree of fulfillment of these conditions determines the degree of market efficiency, which means that in reality we can not talk about full efficiency of capital markets, but only about incomplete and partial effectiveness (Gurgul, 2006, p. 15).

Mature capital markets are characterized by a significant asymmetry of information. The real and relative heihg of transaction costs for institutional and individual investors is also different - they are not defined on equal levels. In addition, the assumption of universal recognition of the significance of the published information and an immediate and unambiguous reaction to them seems to be a contractual assumption for the good of the model, which has little to do with reality.

Immediate reaction, according to E. Peters, also leaves much to be desired. In his opinion, the information reaches the investor in small portions, which he usually does not respond to. Only exceeding a certain critical value triggers a reaction, almost always excessive, compared to recently received information. Immediateity is synonymous with a linear response, information appears and the investor reacts to it.

The linear approach also determines the relationship in which the sum of responses to information is equivalent to the response to the sum of information (Wierzbicki, 2009). If the system (capital market) is non-linear, then its analysis using linear models will be the most significant approximation, and never the exact forecast (Gabryś, 2008, p. 220).

The efficient market hypothesis also assumes that subsequent price changes are independent of each other. The unavoidable effect of the independence of price changes is the pursuit of the distributions of several increments to the distribution consistent with the normal distribution (Kamiński and Komorowski, 2010, p. 15). This means that if subsequent price changes were independent of each other, then the distribution of several of increases would be normal (Wierzbicki, 2009, p. 3).

However, this is not, as confirmed by the Shapiro-Wilk tests published in the literature, as well as skewed and flattened graphs analysis. Lack of rules in this area may also be a testimony of an uneven inflow and various interpretations of information. It should also be noted that if stock prices change independently of earlier quotations, then it is possible to talk about random walk (Bachelier, 1900;

Kendall and Hill, 1953; Osborne, 1959) and construct models of random price wandering, although the efficiency theory itself does not need to be met by prices assumptions about random walk.

The efficient market hypothesis is in three varieties, which assume different types of information, which may be reflected in the prices of securities (Fama, 1970; 1991). Weak Form Efficiency (WFE) hypothesis assumes that prices reflect all information from the past. The implication of this form of EMH is the inability to forecast securities prices, based on their historical development, and therefore on a technical analysis using charts developed on the basis of past quotations of share prices (Peters, 1999, p. 19).

Therefore, it is not possible to create an investment strategy that brings above average returns, based on time series, econometric models, financial models of neural networks or models of chaos theory (Ślepaczuk, 2006, p. 2). In addition, the WFE theory itself has some contradiction: investors do not make decisions solely on the basis of historical information, but the main area of their interest is information about the future.

Semi-Strong Form Efficiency - SSFE assumes that the prices of securities reflect all publicly available information, and therefore information that can be read from a series of time, as well as those contained in current and periodic reports, financial statements of companies and from other sources that may affect the price of shares. E.F. Brigham points out that the acceptance of this form of market efficiency is the fact that stock prices can not be predicted by technical nor fundamental analysis (Brigham, 1996, p. 310).

J. Tobin noted, however, that even if the market is efficient in its medium form and immediately includes all publicly available information, this does not mean that prices reflect the fundamental value as the present value of future cash flows (Tobin, 1967, p. 56). The author distinguished between information market effectiveness and the fundamental market effectiveness. The latter is a narrower approach to the first and assumes that prices only reflect information related to the fundamental value of a given security[3] and P/E ratio, while (Banz, 1981, pp. 3-18) added that this ratio is even higher with companies with lower capitalization. Thus the above researches prompted the rejection of the SSFE hypothesis.

Strong Form Efficiency - SFE assumes that the market is efficient when all information, both publicly available and publicly unavailable and confidential, is

[3]*The most interesting works, examining the SSFE f.e.: (Ball, 1978; Banz, 1981, pp. 3-18; Basu, 1977, pp. 663-682; Fama et al., 1969, p. 10; Fama and French, 1988, pp. 246-273; Kester, 1990; Patell and Wolfson, 1984, pp. 223-252; Rendleman Jr, Jones, and Latane, 1982; Watts, 1978).*

reflected in the price of the security. Therefore, future price development can not be predicted, even with information unavailable to others. In this situation, any analysis of the information loses meaning. Jaffe's (Jaffe, 1974) work on the use of confidential information prior to the announcement of stock split, dividend increase and merger of companies are the examples of a study on the strong efficiency of the market.

The research indicated the possibility of achieving significant profits using confidential information. A similar study was carried out by Friend (Friend, Brown, Herman, and Vickers, 1962; Cornell and Roll, 1981, pp. 201-216; Fama, Fisher, Jensen, and Roll, 1969). The study of strong efficiency was also carried out for the Polish capital market (Czekaj, Woś, and Żarnowski, 2001). Czekaj and others have studied f.e. selection skills and market sens of fund managers operating on the Polish market. In the summary, they claimed that even the knowledge possessed by professional managers does not allow to generate above-average profits, both through skillful selection of shares to the portfolio and forecasting the general economic situation.

The empirical verification of the described hypotheses, which was the result of intensified discussions on the EMH, seems to be more intriguing than the theoretical approach to the problem.

According to R.A. Haugen's degree of market efficiency can be easily verified by checking whether the following conditions are met in a given market (Haugen and Pająk, 1996):

1. New information immediately affects prices that change in a direction consistent with the nature of the information.
2. Changes in securities prices are purely random.
3. When applying any of the available investment strategies from simulation experiments, it is not possible to achieve above-average rates of return.
4. Even professional investors are not able to achieve above-average profits.

If empirical research does not give grounds to reject the hypothesis that the market meets the 1st and 3rd condition, then it means that the market is efficient in a weak form. If there are no grounds to reject the hypothesis that the market meets conditions 2 and 3, then it is efficient in the semi-strong sense. If it is not possible to reject condition 4, then we are dealing with an efficient market in a strong form.

The analysis carried out during the studies verifying the EMH allowed to identify the most frequent deviations from the hypothesis - the so-called capital market anomalies, e.g. underestimation and overestimation of information related to financial results of companies (Abarbanell and Bernard, 1992), financial forecasts (Ball, 1978; Randuman, Jones, and Latare, 1982; Watts, 1978) or dividends (Michaely, Thaler, and Womack, 1995), window dressing effect, January effect

(Gultekin and Gultekin, 1983; Haugen and Lakonishok, 1988; Kato and Schallheim, 1985; Keim, 1983), the Monday effect (French, 1980; Gibbons and Hess, 1981) or anomalies resulting from the psychological aspects of the investment process.

The theory of capital market information efficiency has important implications for the participants of this market, putting into question the legitimacy of technical analysis (weak form efficiency) or fundamental analysis (semi-strong efficiency). However, there is no doubt that abandoning any analysis and resigning from searching for information will paradoxically lead to market inefficiencies (Gabryś, 2008, p. 492). If we assume that the market is information effetive, then by definition, shares can not be underestimated or overestimated. It follows that the cost of capital acquisition by the company is always priced optimally, regardless of the period of bullish or bear market (Gabryś, 2008).

In addition, the appearance of press releases about the company also loses in importance, because if they are based on publicly available data, they will not have any impact on the price, because all information has already been included in it (Szyszka, 2003, p. 35). EMH leads to the conclusion that investors are not able to achieve above-average rates of return on the capital market, which would result from the information they have.

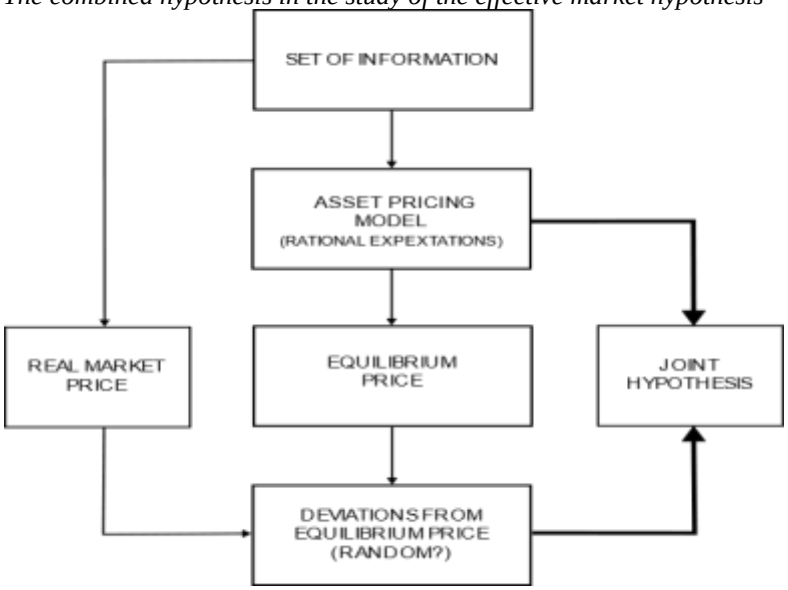
However, as A. Gabryś points out, in modern finance science, it is assumed that the markets are neither perfectly efficient nor completely inefficient and all markets are efficient in relation to a specific group of investors (Gabryś, 2008, p. 503). It seems that this statement is obvious, because in reality investors are heterogeneous. They have various financial resources that they intend to spend on stock market investments, advanced analytical and executive facilities. At the same time, their investment horizons range from a few hours, in the case of day traders, a few days in the case of speculators, to long-term investment strategies of investment funds.

The natural consequence of this state of art is to make different decisions at different times. Even if we assume that all investors have made a decision to buy at the same time, the decision to end the investment will be dictated by the individual selection. In addition, the capital market anomalies are the best confirmation that a specific set of information is not included into the price of securities or is reflected in it with a significant delay. This creates opportunities for investors to achieve above-average rates of return, based on these information.

The methodology of testing EMH is based on two foundations, that were finally formulated by Fama (Fama, 1991) in the joint hypothesis:

1) the asset valuation model,
2) random fluctuations of real prices from the model valuation.

Figure 1. *The combined hypothesis in the study of the effective market hypothesis*



Source: *Own research.*

The information entering the market is processed in the asset valuation model that generates theoretical market equilibrium prices. As far as information is effectively processed in prices, deviations of actual prices from model prices of equilibrium are attributed to random factors. The randomness of price deviations (analyzed by statistical and econometric methods) proves the correctness of the asset valuation model and the inclusion of information available on the market (combined hypothesis).

Due to the combined nature of the hypothesis, problems with the randomness of price deviations from the model equilibrium prices can be attributed either to market information ineffectiveness or incorrect specification of the asset valuation model. This arises a question whether the hypothesis of an efficient market is a falsifiable hypothesis.

In EMH research there is an evolution of the valuation model from naive valuations to complex models based on the portfolio theory. It followed new theoretical results (especially (Markowitz, 1952, pp. 77-99; Sharpe, 1964, pp. 425-442)) and increased computational power. We distinguish two stages of asset valuation modeling:

1) naive stage with random pricing,
2) mature stage in which valuation models are based on the portfolio theory.

In the naive stage, the equilibrium prices were initially modeled with a constant trend. The observation that prices tend to have a different trend than fixed (Timmermann and Granger, 2004, p. 17) quickly led to the adoption of a valuation model with a constant expected rate of return.

In the mature stage, additional factors shaping the rate of return were introduced to the asset valuation model: risk-free rate of return, the market premium and the measure of systematic risk β (Markowitz portfolio theory and the CAPM model). Modern models of asset valuation are based on the assumption that information is available to investors who use it correctly. According to Fama (Fama, 2013, p. 367), testing such valuation models is also testing the hypothesis of an efficient market.

Fama pointed out that the crowning of asset price modeling is the three-factor valuation model. With French (Fama and French, 1993, pp. 3-56), they included to CAPM model variables depending on the capitalization of the company and the ratio of book value to market value, which indirectly introduced the element of fundamental analysis to the valuation.

The valuation model of assets assumed in the efficient market hypothesis is identified with the rational investors' expectations. Investors are analytically transforming held information into the expected rates of return. When the conversion process is over, the rational expected rate of return is not changed, and the actual deviations from it are random until the new information appears. The rationality of investors in the context of expectations is not the neoclassical rationality of investors (homo economicus), defined as maximizing profit. In finance, a rational investor evaluates assets using the discounted cash flow (DCF) method, which is the basis of modern finance theory.

In an article from 1965, P. Samuelson mentioned, as one of the variants proving random walk of prices (proof of martingale), determining the future price, including the discounting of information at the current moment (Samuelson, 1965, pp. 46-47). In 1973, he developed the idea of discounting in the proof of randomness of price change, postulating that the price should be determined using the DCF method based on the expected dividends (Samuelson, 1973, pp. 369-374).

Using discounting future dividends at the moment of price fixing, (Shiller, 1981, pp. 421-436) undermined the hypothesis of the efficient market, showing that the actual prices have excessive volatility compared to the theoretical and rational valuation model of the DCF method. L. Summers (Summers, 1986, pp. 591-601) applied the DCF method in researching the power of market efficiency tests, equating the rationality of investors' expectations with price as the current value of expected cash flows. The DCF valuation model was also used by S. LeRoy (LeRoy, 1989, pp.

1583-1621) in his research on the EMH. In the presented works, the authors used DCF for discrete time, based on variables from the past for the immediate market. In the 1990s, psychological models strongly developed in the science of finance (behavioral economics), and the DCF method ceased to be perceived as giving hope for valuable results (Shiller, 2003, p. 90).

Cash flows in the DCF method are expected. Based on the expected net cash flows, we can determine the expected price of assets in the future, and not just on the current moment, as usually it is assumed in finance (Samuelson, 1937, p. 470). Fama i French (Fama and French, 1988, pp. 55-73) believe that the dependence of current prices on expected prices on the forward market is unique and occurs only on the commodity market.

3. Model of Asset Valuation by Rational Investors

We assume that investors valuing assets behave rationally in neoclassical terms and use the DCF method. It is based on the net cash flows, and these are the result of a fundamental analysis. We will examine the impact on EMH's acceptance that the investor is rational, and not only that he uses rational expectations.

The valuations generated by the presented valuation model are treated as a model of rationality. In the economic reality there are deviations from rationality, which is explained by the psychology of human behavior that deviates from the homo economicus paradigm (behavioral economics). The subject of our further research will be real deviations from the rational trend, which we will try to explain by:

1. Reinterpreting the parameters and variables of the originally defined model.
2. Corrections of the model consistent with the correspondence principle

Let *I(n)* is the information resource at the moment n. The information resource is decoded (transformed) by the investor *i* on net cash flow *N*:

$$\bigwedge_t N_i(I(n), t) = N_i(n, t) \tag{1}$$

and on the discount rate *r*:

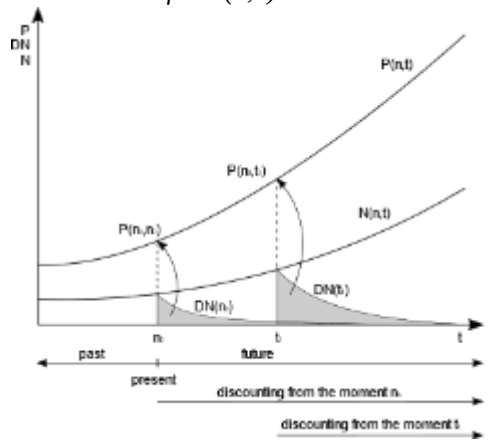
$$\bigwedge_t r_i(I(n), t) = r_i(n, t) \tag{2}$$

For the past and present, when $t \leq n$ net cash flows N decoded from information (I) are known, and the discount rate r can be estimated. In addition, the market price $P(n, t)$ is known for the past and present. For the future, when $t > n$, net cash flows and the discount rate are expected. For each moment in the future, that is for $t > n$, a rational investor sets the expected price $P_i(n,t)$ using the DCF method:

$$P_i(n,t) \stackrel{\text{def}}{=} \int_t^{\infty} N_i(n,\tau) e^{-(\tau - t) r_i(n,t)} d\tau \qquad (3)$$

In formula (3), the price P is a function of time. At any time t of the future, the net cash flows N are discounted on the discount rate r, based on the information gathered from the moment t until moment n. In the formula it is symbolized by the parameterization of the integral (lower limit of integration t) and shiffting the beginning of discounting from the moment t (τ-t exponent). The formula contains three time variables: n, t and τ. The variable n is the moment the information has been collected. The variable t is the moment of time at which we set the price P, and τ is the time of each discounting. Each discounting starts from a given moment t (discount window).

***Figure 2**. Determination of the price P (n, t) in the future based on the expected net cash flows N (n, t) discounted to the value of DN (n, t)*



***Source:** Own rersearch.*

After multiplying N by the discount factor $e^{-r(\tau-t)}$ we get discounted net cash flows (DN). The price P is the sum of the DN (area under the DN function).

The sale and purchase orders are placed on the market with different expected prices $P_i(n, t)$, where i is the index of the valuation formulated by the investor based on the N and r forecasts. Confrontation of supply and demand leads to establishing market

prices on the immediate and forward market. The market price $P\ (n,\ t)$ is the average of prices $P_i\ (n,\ t)$ weighted by the value of the orders executed:

$$P(n,t) = \sum_i w_i(t) P_i(n,t)$$
$$where \bigwedge_t \sum_i w_i(t) = 1 \tag{4}$$

Weights $w_i(t)$ are the function of time. A given investor makes a transaction on selected instruments on the immediate ($n = t$) and forward ($t > n$) market. The larger the investor's order, the greater the relevant weight and the greater impact on the market price. When an investor does not make transactions on a given moment, his weight for this moment is zero.

As a result, based on the time-varying information $I\ (n)$ flowing to investors until the moment n, the asset valuation model by rational investors generates a time-variable price of the asset $P(n,\ t)$ for each moment in the future (for $t > n$).

4. Application of the Asset Valuation Model by Rational Investors

Let's use the model of asset valuation by rational investors to determine share prices in the future based on the assumptions of the Gordon-Shapiro model (increase in the dividend with a constant rate of g) (Gordon, 1959, pp. 99-105; Gordon and Shapiro, 1956, pp. 102-110). We generalize the dividend to net cash flows[4] of N (converted into shares) and we assume that the investor, based on the possessed information, expects future changes in net cash flows with a constant rate of g:

$$N(t) = N_0 e^{gt} \tag{5}$$

where: N_0 - net cash flows in the present (beginning of the analysis), g - constant rate of changes in net cash flows.

With a constant discount rate r, the forward price $P(t)$ according to the Gordon-Shapiro model is equal to:

$$P(t) = \frac{N_0}{r - g} e^{gt} \tag{6}$$

We prove the formula (6) by substituting (5) with the formula (3):

[4]*Net cash flow differs from the dividend: retained earnings, depreciation and changes in, fixed capital, working capital and debt.*

$$P(t) = \int_t^{\infty} N_0 e^{g\tau} e^{-r(\tau - t)} d\tau = N_0 \int_t^{\infty} e^{rt-(r-g)\tau} d\tau$$

$$= N_0 \lim_{b\to\infty} \int_t^b e^{rt-(r-g)\tau} d\tau \underbrace{=}_{\substack{\varphi = rt-(r-g)\tau \\ d\varphi = -(r-g)d\tau \\ d\tau = \frac{d\varphi}{-(r-g)}}} N_0 \lim_{b\to\infty} \int_t^b -\frac{e^{\varphi}}{r-g} d\varphi \quad (7)$$

$$= \frac{N_0}{r-g} \lim_{b\to\infty} -e^{\varphi} \Big|_t^b = \frac{N_0}{r-g} \lim_{b\to\infty} e^{rt-(r-g)\tau} \Big|_b^t$$

$$= \frac{N_0}{r-g} \lim_{b\to\infty} \left(e^{rt-(r-g)t} - e^{rt-(r-g)b} \right)$$

$$= \frac{N_0}{r-g} \left(e^{gt} - \lim_{b\to\infty} e^{rt-(r-g)b} \right) = \frac{N_0}{r-g} e^{gt}$$

The *P* price in the future according to the Gordon-Shapiro model grows exponentially with the growth rate *g*, i.e., the growth rate of net cash flow *N*. In the special case, when *t = 0* (current valuation), we get the formula given by Gordon and Shapiro:

$$P(0) = \frac{N_0}{r-g} \quad (7)$$

It can be concluded that the Gordon-Shapiro model is a special case of the valuation model we proposed.

The total rate of return *TR* according to Gordon-Shapiro model is constant in time:

$$TR = r \quad (8)$$

The equality of the total rate of return and the discount rate (constant by definition) in the Gordon and Shapiro models will be proved by starting from the definition of the total rate of return *TR* as the sum of the rate of return on capital and net cash flow.

From the definition of *TR* and (5) and (6) we get:

$$TR \stackrel{\text{def}}{=} \frac{d \ln P}{dt} + \frac{N}{P} = \frac{d \ln \frac{N_0}{r-g} e^{gt}}{dt} + \frac{N_0 e^{gt}}{\frac{N_0}{r-g} e^{gt}}$$
$$= \frac{d \ln \frac{FCF_0}{r-g}}{dt} + \frac{d \ln e^{gt}}{dt} + (r-g) = 0 + \frac{dgt}{dt} + (r-g)$$
$$= g + (r-g) = r \quad (9)$$

We have proved that on the basis of information about the current value of net cash flow N_0 and the rational investor's expectation that in the future they will be in line with the trend of a constant rate of change g, the forward price of shares will change exponentially with a constant rate g, and the total rate of return will be constant and equal to the discount rate. Therefore, the model with a constant rate of change in net cash flows is a model that meets the assumptions of these EMH tests that examine the model at a constant total rate of return (*TR*).

A rational investor predicts forward prices, based on the information *I(n)* available at the present moment n concerning:

1) the present - in the Gordon-Shapiro model the initial value of net cash flows N_0,
2) expected future conditions - in the Gordon-Shapiro model, the net cash flows are in line with the exponential trend with a constant expected rate of change *g*.

Thanks to the DCF method, a rational investor can determine the expected price on the basis of the above information at any time in the future - in the Gordon-Shapiro model, prices shaped according to the exponential trend.

Each defined by a given rational investor *i* the $P_i(n, t)$ price is based on information about the present and future available until the moment *n*. The prices $P_i(n, t)$ expected by investors are aggregated in the market equilibrium process into market prices *P(n,t)*. Thus, the market price *P(n, t)* is shaped by the information resource *I(n)* interpreted by rational investors**.** Market prices *P(n, t)* contain information about the past, present and expectations.

We can specify the following assignments:

$I(n) \rightarrow N_i(n, t),\ r_i(n, t) \rightarrow P_i(n, t) \rightarrow P(n, t)$

Where:

1) $I(n) \rightarrow N_i(n, t),\ r_i(n, t)$ is the process of decoding information,
2) $N_i(n, t),\ r_i(n, t) \rightarrow P_i(n, t)$ is the DCF method of valuation,

3) $P_i(n, t) \rightarrow P(n, t)$ is the process of balancing the market and determining the market price.

It is not possible to assign the opposite to the above. First of all, on the basis of $P_i(n, t)$ prices, you can not determine $N_i(n, t)$ and $r_i(n, t)$, because the result in the DCF method contains the sum of discounted cash flows, devoid of information on the distribution of these flows, and has not enough degrees of freedom for the reproduction of variables N i r.

Moreover, based on market prices $P(n, t)$ it is difficult to determine the prices $P_i(n, t)$ expected by individual investors who placed orders on the market (ignorance of weights). In the process of determining the market price, information about the fundamental basis of DCF prices is "lost". At the market level, we know that expected prices in the future are based on expectations of fundamental information, but we do not know what they are. Efficient market hypothesis tests using prices (rates of return) analyze the information effectiveness that is impoverished in relation to that used by a rational investor.

Some tests of the efficient market hypothesis go even further in narrowing down the data set, limiting itself to the analysis of prices (rates of return) from the past and present. They omitted the expected prices (rates of return) in the future, which means that the fundamental variables expected by investors are omitted in the research process. This is tantamount to rejection of the investor's rationality principle. Of course, every past price includes some information about the current future (due to discounting to infinity). However, from the present point of view, the information is outdated because it comes from the past, from the time when the past price was determined.

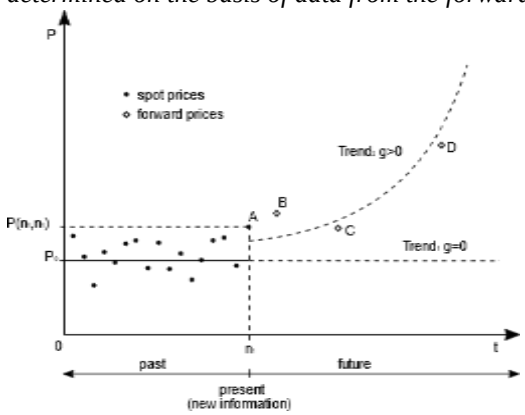
EMH is interpreted so that information arriving on the market is quickly and fully taken into account in current prices (Fama, 2013, p. 368). Let us assume that at the moment n the investor acquires information that the company has launched more efficient production of goods that better satisfying a consumer's need, as a result of which the investor expects the rate of changes in net cash flows to grow from $g + \Delta g$, where $\Delta g > 0$. Then in accordance with (6) the price of the company's shares in the future for $t > n$ will be:

$$P(t) = \frac{N_0}{r - g - \Delta g} e^{(g+\Delta g)t} \tag{10}$$

The equation (11) shows that the current price of the shares (for $t = 0$) will increase, but at the same time the share prices will increase in the future. The increase in share prices in the future will be caused not only by the increase in the current price, but also as a result of the increase in price dynamics changes. Therefore, information can affect not only the current price, but also prices in the future.

Let us assume that the theoretical price model is correct and determined by the equation (6) $P(t) = P_0 e^{gt}$, where $P_0 = N_0 /(r - g)$. By correctness, we mean the stationarity of deviations of the time series of prices from the trend (random price fluctuations around the trend). Let us assume that initially the price trend P before disclosure is constant (Figure 3, $Trend_1$ $g = 0$).

Figure 3. *Change in the price trend P after disclosure at the time of n_1 new information, determined on the basis of data from the forward market.*



Source: *Own study.*

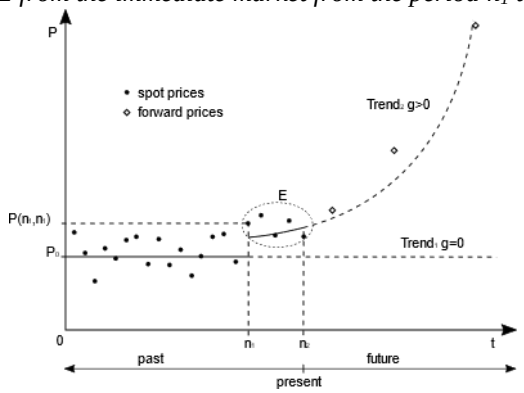
At n_1, investors get information, they correctly (in the above approach) interpret it: the expected future net cash flow growth will increase to $g > 0$ ($Trend_2$). According to (6) investors expect that stock prices will also increase at a constant rate g. At the same time, the current price A at the moment n1 on the immadiate market will probably be fixed (price is a random variable) at a higher level than resulting from the current constant trend ($Trend_1$). Expected prices on the forward market (B-D) with the A-price will be scattered around the new exponential trend.

Let's now take the perspective of the researcher. Although the current price A takes into account the new information, the researcher analyzing only prices from the immediate market will treat the deviation of this single price from the current trend (in our case, $Trend_1$) as incidental (random) and loses the possibility of determining that the price includes new information. Taking into account the prices of B-D from the forward market will allow the researcher to assess that the price A does not oscillate in the current (constant) trend, but around the new trend (exponential $Trend_2$).

All deviations of A-D prices from the constant trend are in fact more numerous and with consequences in statistical analysis (eg autocorrelation) than a single deviation of the price A. The conclusion about the emergence of a new trend will gain a higher

probability as a result of the analysis for the next moment n_2, when the new trend may be designated not only on the basis of prices from the forward market, but also prices from the immediate market E quoted from n_1 to n_2 (Figure 4).

Figure 4. *Making the conclusion about the appearance of a new price trend P after considering the price E from the immediate market from the period n_1 to n_2*



Source: *Own study.*

The above analysis is generalized for any functions, correctly established for past data $P_P(t)$ and new $P_F(t)$ function correctly set for the current price and prices from the forward market after the appearance of new information resulting in non-random deviations of the current and forward prices from the trend determined by the function $P_P(t)$.

In the described situation, autocorrelation of current price deviations and prices on the forward market from the current trend since the disclosure of new information does not prove the informational inefficiency of the market. On the contrary, it proves that the market reacted correctly to new information. This requires making prices from forward market the subject to examination. In the situation of the volatility of information affecting the volatility of the price model specification used in testing the effective market hypothesis, you can not omit the forward market data at the risk of making mistake and recognize that the market is informatively ineffective, despite its actual information efficiency.

5. Discussion

A rational investor in determining the trend of spot and forward prices uses all available information. The observation of Fama and French about the lack of influence of forward prices on the current price leads to the following hypotheses:

1) The cause of the anomaly is a methodological problem,
2) Investors are irrational.

The prices expected in the future have a greater variance than other prices (past spoot prices), because the variance of forward prices is higher by prediction errors. Prediction errors may overlap from moment to moment, which in statistical terms leads to random walk. In this case, the variance of forward prices increases as the horizon of the prediction increases. As a consequence, the high and growing noise accompanying forward prices significantly impedes the estimation of the expected value (trend). This may result in a conclusion about the statistically insignificant impact of forward prices on current prices.

The rejection of forward prices in the analysis leads to the limitation of the subject of research. Only the spot market becomes the subject of them, ignoring the evaluation of the forward market efficiency. In an extreme case (the past minus the infinity, the future plus the infinity, or the equality of the period from which past data are derived and the period in which forward prices are quoted) means a shortening of the period from which the prices are realized by half. The statistical effect of sample shortening is to filter out a possible long-term trend.

In particular, when prices change cyclically, the shortening of the sample by half results in the failure to notice a trend around which there are fluctuations of a period equal to twice the time from which the sample originates. The elimination of the realisation of prices from the forward market distorts the shorter trend estimated on the basis of spot prices, especially as regards the recent prices. A distortion of the short-term trend leads to a change in the characteristics of the random component. In particular, the autocorrelation of the random component is changed. These changes increase the risk of error in EMH testing.

6. Summary

The tests of the efficient market hypothesis, according to the principle of investor rationality, should take into account the future data held by investors, for example included in forward market prices, or published forecasts of fundamental values. EMH testing based on prices, rates of return and other data from the past and present is burdened by the omission of information available to a rational investor about the future.

The obtained results inspire further research into the trends of stock prices and the characteristics of the random noise of prices which are of a rational nature. Deviations from the rational model may constitute a measure of the investor's irrationality and market information inefficiency. This will be the subject of subsequent publications.

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