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A Heavy Weighted WASPAS Approach and Its Application into Multi-Attribute Choice Problems

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Abstract:

Purpose: *The purpose of this paper is to propose a novel multi-attribute decision-making (MADM) method, referred to as the Heavy Modified Weighted Aggregated Sum Product Assessment (WASPAS-HM), which introduces significant enhancements to the original cardinal ranking WASPAS method developed in 2012, particularly for screening decision problems involving large sets of alternatives.*
Design/Methodology/Approach: *The proposed WASPAS-HM method incorporates several methodological modifications, including: (1) the ability to handle ordinal, cardinal, or mixed attribute evaluations; (2) the use of meaningful monotonic normalization techniques; (3) heavy aggregation of normalized attribute values; (4) safe screening procedures for identifying a top subset of promising alternatives; and (5) the re-scaling of heavy additive and multiplicative relative importance for the screened alternatives. The method employs heavy weighted average (HWA) and heavy weighted geometric (HWG) aggregation operators in place of the weighted sum model (WSM) and weighted product model (WPM) used in the original WASPAS framework.*
Findings: *The study introduces the heavy modified WASPAS (WASPAS-HM) screening method as an effective tool for solving complex multi-attribute choice problems. The results demonstrate that the method can robustly accommodate different types of evaluation data while improving ranking stability and discrimination power compared to the original WASPAS approach.*
Practical Implications: *The proposed method is particularly suitable for decision-making contexts involving a large number of pre-specified alternatives, where preliminary screening is essential. A worked numerical example is presented to demonstrate the applicability, relevance, and computational accuracy of the WASPAS-HM method.*
Originality/Value: *This research extends the WASPAS methodology by introducing heavy aggregation operators and a structured screening mechanism, thereby enhancing its flexibility and effectiveness in large-scale MADM problems. The WASPAS-HM method represents a valuable contribution to the decision sciences literature by offering a more robust and adaptable ranking and screening framework.*

Keywords: *Multi-attribute decision making, Heavy aggregation, Screening, Heavy weighted operators, WASPAS.*

JEL Classification: *C44, C60, D81, D70, C02.*
Paper type: *Research article.*

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1. Introduction

In the present work, we deal with the fundamental multi-attribute decision making (MADM) problem of selecting a 'best' alternative form among a finite, given and fixed choice set (i.e., the set of alternatives available). Or, to solve the above multi-attribute choice problem, most of the MADM methods proposed till now in academic literature adopt the 'rank-then-select' view.

The ranking-based view states that at first a ranking of competing alternatives from most preferred to least preferred (possibly with ties) is obtained, then, an alternative that is ranked highest is chosen. An alternative, totally different, view is 'screen-then-select'. The screening-based view states that at first the alternatives which are unlikely to be chosen are screened out, then, after that the ‘to be selected" alternative is identified among the screened top subset of alternatives.

The main point of this contribution is to propose a *Heavy Modified Weighted Aggregated Sum Product* method of screening to winnow out the competing alternatives leaving only the promising ones. And, herein, we refer to this method as WASPAS-HM.

Note that the original WASPAS [1, 3-6, 10, 14, 16, 21, 22] is a MADM method which rests on the ranking-based view. This method combines the weighted sum model (WSM) and the weighted product model (WPM). Or the main drawback of the WASPAS method is that it is purely cardinal and thus the normalization techniques used in this method are without doubt *questionable* in ordinal or mixed evaluation contexts. The suggested WASPAS-HM method deviates from the classical WASPAS method in different ways. In fact, in the WASPAS-HM method, one is allowed:

- To use ordinal, cardinal or mixed evaluations.
- To perform meaningful monotonic normalization of attribute values.
- To assign heavy weightage to attributes.
- To execute screening to whittle down the number of alternatives.
- To rescale the heavy relative importances of screened alternatives.

In the remainder of this paper, we will (1) recall the classical WASPAS method, (2) describe the suggested WASPAS-HM method, (3) provide a worked example, and (4) end the paper with some concluding remarks.

2. The Classical WASPAS Method

2.1 Basic problem

The basic problem at hand is as follows:

Given:

1. m pre-specified competing alternatives $\{A_i\}_{i=1}^{i=m}$.
2. n cardinal evaluation attributes $\{C_j\}_{j=1}^{j=n}$.
3. A decision matrix, $[x_{ij}]$, where x_{ij} denotes the evaluation of alternative A_i with respect to attribute C_j (i = 1 to m and j = 1 to n).
4. A set $w = \{w_j\}_{j=1}^{j=n}$ of n attribute weights satisfying $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$.

Goal:

To rank the alternatives from most preferred to least preferred.

2.2 Normalization

As pointed out earlier, the original WASPAS method uses cardinal evaluations. And the normalization techniques used in this method are the following:

i) For a benefit attribute, the normalized attribute value x_{ij}^N will be given by:

$$x_{ij}^N = \frac{x_{ij}}{x_j^{max}} \tag{1}$$

ii) For a cost attribute, the normalized attribute value x_{ij}^N will be expressed as:

$$x_{ij}^N = \frac{x_j^{min}}{x_{ij}} \tag{2}$$

In Eqs. (1) and (2) above, x_j^{max} denotes the maximum attribute value with respect to attribute C_j $(j = 1\ to\ n)$, and x_j^{min} the minimum attribute value.

Next, the WASPAS method defines the additive relative importance, denoted as $Q_i^{(1)}$, and the multiplicative relative importance, denoted as $Q_i^{(2)}$, of alternative

$A_i (i = 1\ to\ m)$, respectively, as

$$Q_i^{(1)} = \sum_{j=1}^{n} w_j\, x_{ij}^N \tag{3}$$

$$Q_i^{(2)} = \prod_{j=1}^{n} (x_{ij}^N)^{w_j} \tag{4}$$

The joint generalized criterion value, denoted as Q_i, of alternative $A_i (i = 1 \; to \; m)$ will then be defined by:

$$Q_i = \lambda \times Q_i^{(1)} + (1 - \lambda) \times Q_i^{(2)} \tag{5}$$

where the parameter $\lambda \in [0, 1]$.

Finally, the alternatives are ranked in descending order of their joint generalized criterion values. We now turn to the proposed WASPAS-HM method.

3. The WASPAS-HM Method

To begin, we present some basic definitions, notations and formulas which will be used to elaborate the proposed method. We will present in turn meaningful normalization for ordinal, cardinal and mixed evaluations, the heavy aggregation operators involved, the screening strategies and the choice rule.

3.1 Meaningful Monotonic Normalization

As already said, the WASPAS-HM method uses ordinal, cardinal or mixed evaluations. Therefore, we will present adequate monotonic normalization procedures for these types of evaluations.

3.1.1 The case of ordinal attribute values

For any ordinal attribute C_j, the normalized attribute value x_{ij}^N will be expressed as:

$$x_{ij}^N = \frac{m_{ij}^+ - m_{ij}^- + m - 1}{2(m-1)} \tag{6}$$

where m_{ij}^+ is the number of alternatives beaten by alternative A_i and m_{ij}^- the number of alternatives beating A_i [11].

3.1.2 The case of cardinal values

For any benefit attribute C_j, the normalized attribute value x_{ij}^N will be given by:

$$x_{ij}^N = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}} \tag{7}$$

and for any cost attribute C_j, the normalized attribute value x_{ij}^N will be expressed as:

$$x_{ij}^N = \frac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}} \tag{8}$$

In the Eqs. (7) and (8) above, x_j^{min} denotes the minimum attribute value with respect to attribute C_j $(j = 1 \ to \ n)$, and x_j^{max} the maximum attribute value.

The above monotonic normalization is meaningful in the sense of measurement theory [2].

3.2 Aggregation Operators

In most multi-criteria decision making (MCDM) models, assigning weights to criteria is an important step that needs to be reexamined [24]. Though, determining the weights of criteria is one of the key problems that arise in multi-criteria decision making (Dragan *et al.,* 2018). Considering the fact that the weights of criteria can significantly influence the outcome of the decision-making process, it is important to pay particular attention to the objectivity factors of criteria weights. For this reason, we will use heavy weights to take a large importance of the criteria evaluation.

Hereafter, we will make use of the following two aggregation operators, namely, the heavy weighted average (HWA) operators [12] and the heavy weighted geometric (HWG) operators [20]. By using the HWA operator, we allow the output values returned by this operator to range from the minimum to the total.

Definition 1. [19] A *n*-dimensional vector $W = (W_1, W_2, \dots, W_n)$ is a heavy weighting vector if and only if:

i. $0 \leq W_j \leq 1$

ii. $1 \leq |W| \leq \mathrm{n}$

where $|W| = \sum_{j=1}^{n} W_j$ is the magnitude of *W*.

Definition 2. [12] An *n*-ary heavy weighted average (*HWA*) operator is a mapping *HWA*: $R^n \rightarrow R$ that has an associated *n*-dimensional heavy weighting vector *W* such that

$$HWA(x_1, \dots, x_n) = \sum_{j=1}^{n} W_j \, x_j \tag{9}$$

where x_j represents the *j*th argument variable.

Definition 3. [20] An *n*-ary heavy weighted geometric (*HWG*) operator is a mapping *HWG*: $R^n \rightarrow R$ that has an associated *n*-dimensional heavy weighting vector *W* such that:

$$HWG\,(x_1, \dots, x_n) = \prod_{j=1}^{n} (x_j)^{W_j} \tag{10}$$

Now, if W be a heavy weighting vector, then the suggested WASPAS-HM method defines the heavy additive relative importance, written $HQ_i^{(1)}$, and the heavy multiplicative relative importance, written $HQ_i^{(2)}$, of alternative $A_i\,(i = 1\ to\ m)$, respectively, as

$$HQ_i^{(1)} = \sum_{j=1}^{n} W_j\, x_{ij}^N \tag{11}$$

$$HQ_i^{(2)} = \prod_{j=1}^{n} (x_{ij}^N)^{W_j} \tag{12}$$

Note that if the magnitude $|W| = 1$, then $HQ_i^{(1)} = Q_i^{(1)}$ (additive relative importance) and $HQ_i^{(2)} = Q_i^{(2)}$ (multiplicative relative importance).

3.3 Screening Strategies

By using screening, the finite, given and fixed choice set of multi-attribute alternatives is narrowed down to a manageable top set of promising alternatives. For a deeper understanding of "Screening", the reader can refer to Refs. [7-9, 15, 17, 18].

Herein, the suggested WASPAS-HM method employs this process by putting to use the following two screening strategies:

- Pareto-dominance-based screening.
- Relative-importance-based screening.

The first strategy used is Pareto-dominance-based screening. '*Dropping dominated alternatives is logical because a valid multicriteria method will never choose a dominated alternative*' [9]. The definition of Pareto-dominance is as follows.

Definition 4. An alternative A_k will be said to be *Pareto-dominated* by alternative A_l if and only if:

$$x_{lj}^N \geq x_{kj}^N \tag{13}$$

with at least one strict inequality.

To eliminate further alternatives, we introduce the following notion of *HQ-dominance.*

Definition 5. An alternative A_k will be said to be *HQ-dominated* by alternative A_l if and only if and only if

$$HQ_l^{(1)} \geq HQ_k^{(1)} \text{ and } HQ_l^{(2)} \geq HQ_k^{(2)} \tag{14}$$

with at least one strict inequality.

3.4 Choice Rule

In order to make a final choice among non *HQ*-dominated alternatives, one must choose the alternative A_i with the largest heavy joint generalized criterion value, written HQ_i, given by:

$$HQ_i = \lambda \times RHQ_i^{(1)} + (1-\lambda) \times RHQ_i^{(2)} \tag{15}$$

for $\lambda \in [0, 1]$, where the rescaled values $RHQ_i^{(1)}$ and $RHQ_i^{(2)}$ are given by:

$$RHQ_i^{(1)} = \frac{HQ_i^{(1)}}{\max_k HQ_k^{(1)}} \tag{16}$$

$$RHQ_i^{(2)} = \frac{HQ_i^{(2)}}{\max_k HQ_k^{(2)}} \tag{17}$$

Note that the rescaled heavy relative importances are computed solely for non HQ-dominated alternatives in such a way that the largest values are assigned a value of one.

3.5 Generic Steps

The suggested WASPAS-HM method is meant to help the decision maker reach a final choice. The generic steps of this method are listed below:

Step 1: To *build* an evaluation table.
Step 2: To *normalize* the evaluation table.
Step 3: To *screen out* (if applicable) Pareto-dominated alternatives.
Step 4: To *compute* $HQ_i^{(1)}$ and $HQ_i^{(2)}$ for each non-dominated alternative A_i.
Step 5: To *screen out* (if applicable) *HQ-dominated* alternatives.
Step 6: To *compute* $RHQ_i^{(1)}$ and $RHQ_i^{(2)}$ for each non *HQ*-dominated alternative A_i.
Step 7: To *compute* HQ_i for each non *HQ*-dominated alternative A_i.
Step 8: To *select* an alternative with largest HQ_i value.

In order to show the relevance and accuracy of the proposed method, we will exploit the hypothetical example in Ref. [13]. And we will first identify a 'best' alternative, and then we will compare it with the one obtained by the sophisticated screening and selection method developed by Rebaï and colleagues in Ref. [13].

4. Worked Example

In this example, ten alternatives $\{A_i\}_{i=1}^{i=10}$ are judged on six attributes $\{C_j\}_{j=1}^{j=6}$. The alternatives are rated on an ordinal scale made up of nine discrete steps denoted by G_k (k = 1, 2, . . ., 9) such that $G_{k+1} > G_k$ for k = 1, 2, . . . , 8.

Given the evaluations shown in Table 1 below, which alternative should one choose when applying WASPAS-HM?

Table 1. *Evaluation table*

Attribute	C_1	C_2	C_3	C_4	C_5	C_6
Heavy weight	0.92	0.64	0.41	0.22	1	0.23
A_1	G_1	G_1	G_9	G_5	G_6	G_7
A_2	G_5	G_1	G_7	G_5	G_8	G_7
A_3	G_6	G_1	G_8	G_2	G_7	G_6
A_4	G_3	G_5	G_4	G_4	G_5	G_3
A_5	G_4	G_4	G_6	G_1	G_2	G_1
A_6	G_7	G_6	G_2	G_5	G_1	G_1
A_7	G_2	G_7	G_3	G_6	G_4	G_4
A_8	G_9	G_2	G_1	G_6	G_4	G_5
A_9	G_8	G_3	G_5	G_6	G_3	G_8
A_{10}	G_5	G_4	G_2	G_3	G_1	G_1

Table 2. *Normalized evaluation table*

Alter.	C_1	C_2	C_3	C_4	C_5	C_6
w_j	***0.92***	***0.64***	***0.41***	***0.22***	***1***	***0.23***
A_1	*0*	*0.1111*	*1*	*0.5556*	*0.7778*	*0.8333*
A_2	*0.5*	*0.1111*	*0.7778*	*0.5556*	*1*	*0.8333*
A_3	*0.6667*	*0.1111*	*0.8889*	*0.1111*	*0.8889*	*0.6667*
A_4	*0.2222*	*0.7778*	*0.4444*	*0.3333*	*0.6667*	*0.3333*
A_5	*0.3333*	*0.6111*	*0.6667*	*0*	*0.2222*	*0.1111*
A_6	*0.7778*	*0.8889*	*0.1667*	*0.5556*	*0.0556*	*0.1111*
A_7	*0.1111*	*1*	*0.3333*	*0.8889*	*0.5*	*0.4444*
A_8	*1*	*0.3333*	*0*	*0.8889*	*0.5*	*0.5556*
A_9	*0.8889*	*0.4444*	*0.5556*	*0.8889*	*0.3333*	*1*
A_{10}	*0.5*	*0.6111*	*0.1667*	*0.2222*	*0.0556*	*0.1111*

It is easy to see that A_{10} is Pareto dominated by A_6. Therefore, A_{10} will be screened out.

Next, the heavy relative importances $HQ_i^{(1)\prime}$s and $HQ_i^{(2)\prime}$s of the remaining alternatives will be shown in Table 3.

Table 3. *Heavy relative importances*

Alt.	$HQ_i^{(1)}$	$HQ_i^{(2)}$
A_1	1.5728	0
A_2	2.1639*	0.0984
A_3	2.1156	0.0803
A_4	1.7011	0.0622
A_5	1.2188	0
A_6	1.5562	0.0104
A_7	1.6766	0.0341
A_8	1.9567	0
A_9	2.0889	0.1363*

This time, it is easy to see that the alternatives A_1, A_3, A_4, A_5, A_6, A_7 and A_8 are *HQ-dominated* by A_2.

Therefore A_2 and A_9 are the only non *HQ-dominated* alternatives. Next, we rescale the $HQ_i^{(1)\prime}s$ and $HQ_i^{(2)\prime}s$ by dividing them by their respective maximum. We then get the rescaled heavy relative importances $RHQ_i^{(1)\prime}s$ and $RHQ_i^{(2)\prime}$s shown in Table 4.

Table 4. *Rescaled heavy relative importances*

Alt.	$RHQ_i^{(1)}$	$RHQ_i^{(2)}$
A_2	1	0.72
A_9	0.96	1

Lastly, to make a final choice, we need to compute the heavy joint generalized criterion values HQ_2 and HQ_9.

Table 5. *Impact of the parameter λ on the final choice.*

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
HQ_2	0.748	0.776	0.804	0.832	0.860	0.888	0.916	0.944	0.972	1
HQ_9	0.996	0.992	0.988	0.984	0.980	0.976	0.972	0.968	0.964	0.960

To conclude, one can easily notice that alternative A_2 is the most suitable if $\lambda \in [0.9, 1]$, and that A_9 is to be preferred when $\lambda \in [0.1, 0.8]$.

5. Concluding Remarks

In this contribution, we introduced the MADM method named *heavy modified WASPAS* (WASPAS-HM) method of screening to solve multi-attribute choice problems. This method can handle ordinal, cardinal or mixed evaluations, and it employs, among others, the heavy weighted average (HWA) and heavy weighted geometric (HWG) aggregation operators instead of the weighted sum model (WSM) and weighted product model (WPM) used in the original WASPAS method. In the future, we plan to elaborate the *heavily ordered* modified WASPAS and *induced heavy ordered* modified WASPAS methods of screening.

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